## Bounded-Cost Search Using Estimates of Uncertainty

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Objective: Find a plan with cost at most $C$ as fast as possible.

## Previous Approaches

- Standard Heuristic Search with Pruning on $C$
- Potential Search (PTS) ${ }^{1}$
$\rightarrow$ best-first search on $\frac{h}{C-g}$
- Bounded-cost Explicit Estimation Search (BEES) ${ }^{2}$
$\rightarrow$ focal search:
- open sorted by $f$, only nodes with $g+h \leq C$
- focal sorted by $d$, only nodes with $g+\hat{h} \leq C$ ( $\hat{h}$ is corrected for the observed heuristic error ${ }^{3}$ )

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## Expected Effort Search (XES)

$$
\text { Best-first search on the expected effort: }{ }^{4} \frac{T}{p}
$$

- $T(n)$ : search effort to find a solution under $n$
- $p(n)$ : probability that $n$ leads to a solution within $C$


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(n2) $\begin{aligned} & T=6 \\ & p=0.25\end{aligned}$
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How to obtain $T$ ?
$\rightarrow$ just use $d$
How to obtain $p$ ?
$\rightarrow$ approximate from $\hat{h}$ distribution

## Obtaining $p$ from Belief Distributions

Nancy ${ }^{5}$ belief distributions based on heuristic error observations ${ }^{6}$ :

${ }^{5}$ Mitchell et al. 2019.
${ }^{6}$ Thayer, Dionne, and Ruml 2011.

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## Theoretical Analysis

XES optimizes search effort, assuming

1. the search explores one subtree at a time,
2. subtrees are independent, and
3. subtrees are abandoned after spending $T(n)$ time.

## Using $p$ in BEES

## BEES95:

- open sorted by $f$, only nodes with $g+h \leq C$
- focal sorted by $d$, only nodes with $y+\hat{h} \leq G p(n)>95 \%$


## Experiments

## Planning Domains:

- Implementation in Fast Downward ${ }^{7}$
- Benchmarks:
- IPC'18 cost-bounded track
- Previous IPC domains with bounds from Planning.Domains ${ }^{8}$

Search Domains:

- Sliding-Tile Puzzle, Vacuum World, Pancake, Racetrack

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## IPC＇18 Bounded－Cost Track

| Coverage | $\begin{aligned} & \omega \\ & \stackrel{\omega}{\omega} \\ & 0 \end{aligned}$ | $\stackrel{\curvearrowleft}{\llcorner }$ | $\begin{aligned} & \Omega \\ & 山 山 ⿱ 屮 凵 \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{\sim} \\ & \underset{\sim}{山} \\ & \underset{\sim}{u} \end{aligned}$ | $\stackrel{\sim}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Agricola（20） | 1 | 0 | 0 | 0 | 0 |
| Caldera（20） | 8 | 10 | 10 | 12 | 13 |
| Caldera－split（20） | 4 | 2 | 2 | 2 | 2 |
| DataNetwork（20） | 2 | 0 | 3 | 3 | 4 |
| Nurikabe（20） | 4 | 10 | 10 | 11 | 9 |
| Settlers（20） | 4 | 5 | 10 | 11 | 11 |
| Snake（20） | 4 | 5 | 4 | 4 | 5 |
| Spider（20） | 7 | 11 | 10 | 10 | 9 |
| Termes（20） | 11 | 9 | 11 | 11 | 13 |
| Sum（180） | 45 | 52 | 60 | 64 | 66 |
| Expansions（ $* 10^{3}$ ） | 1.93 | 3.93 | 2.10 | 2.25 | 1.77 |

## IPC'18 Bounded-Cost Track

| Coverage | $\begin{aligned} & \sim \\ & \stackrel{\sim}{\infty} \\ & \hline \end{aligned}$ | $\stackrel{\sim}{\llcorner }$ | $\begin{aligned} & \sim \\ & \sim \\ & \sim \end{aligned}$ | $\xrightarrow[\sim]{\sim}$ | $\stackrel{\sim}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Agricola (20) | 1 | 0 | 0 | 0 | 0 |
| Caldera (20) | 8 | 10 | 10 | 12 | 13 |
| Caldera-split (20) | 4 | 2 | 2 | 2 | 2 |
| DataNetwork (20) | 2 | 0 | 3 | 3 | 4 |
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| Settlers (20) | 4 | 5 | 10 | 11 | 11 |
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$\rightarrow$ Cost-bounded algorithms dominate GBFS; XES is best overall.

## Pre-2018 IPC Domains



## Search Domains: Heavy Vacuum World



## Search Domains

Uniform Tile


## Search Domains

Heavy Tile


## Conclusion

## Expected Effort Search (XES):

- Optimizes search effort in a simplified model.
- Superior robustness and performance to comparable algorithms.
$\rightarrow$ Advances the trend of leveraging distributional information in deterministic heuristic search.


[^0]:    ${ }^{1}$ Stern, Puzis, and Felner 2011.
    ${ }^{2}$ Thayer et al. 2012.
    ${ }^{3}$ Thayer, Dionne, and Rum 2011.

[^1]:    ${ }^{7}$ Helmet 2006.
    ${ }^{8}$ Muse 2016.

