

# Metareasoning for Heuristic Search Using Uncertainty

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Laura Dietz

Levi Lelis

Marek Petrik

Wheeler Ruml (Advisor)

# The Thesis of My Dissertation

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■ Metareasoning

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Online Planning

Bounded-Cost  
Search

Conclusion

heuristic search can benefit from representing uncertainty

scalar heuristic  $\rightarrow$  belief distribution that represents uncertainty

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heuristic search can benefit from representing uncertainty

scalar heuristic  $\rightarrow$  belief distribution that represents uncertainty

show in three problem settings:

- real-time heuristic search
- concurrent planning and execution
- bounded-cost search

# What is Heuristic Search?

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heuristic search: a planning approach

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heuristic search: a planning approach

planning is a model-based AI method, it models the environment as a state space and finds a sequence of actions that accomplishes some objective

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planning is a model-based AI method, it models the environment as a state space and finds a sequence of actions that accomplishes some objective

**heuristic search:**

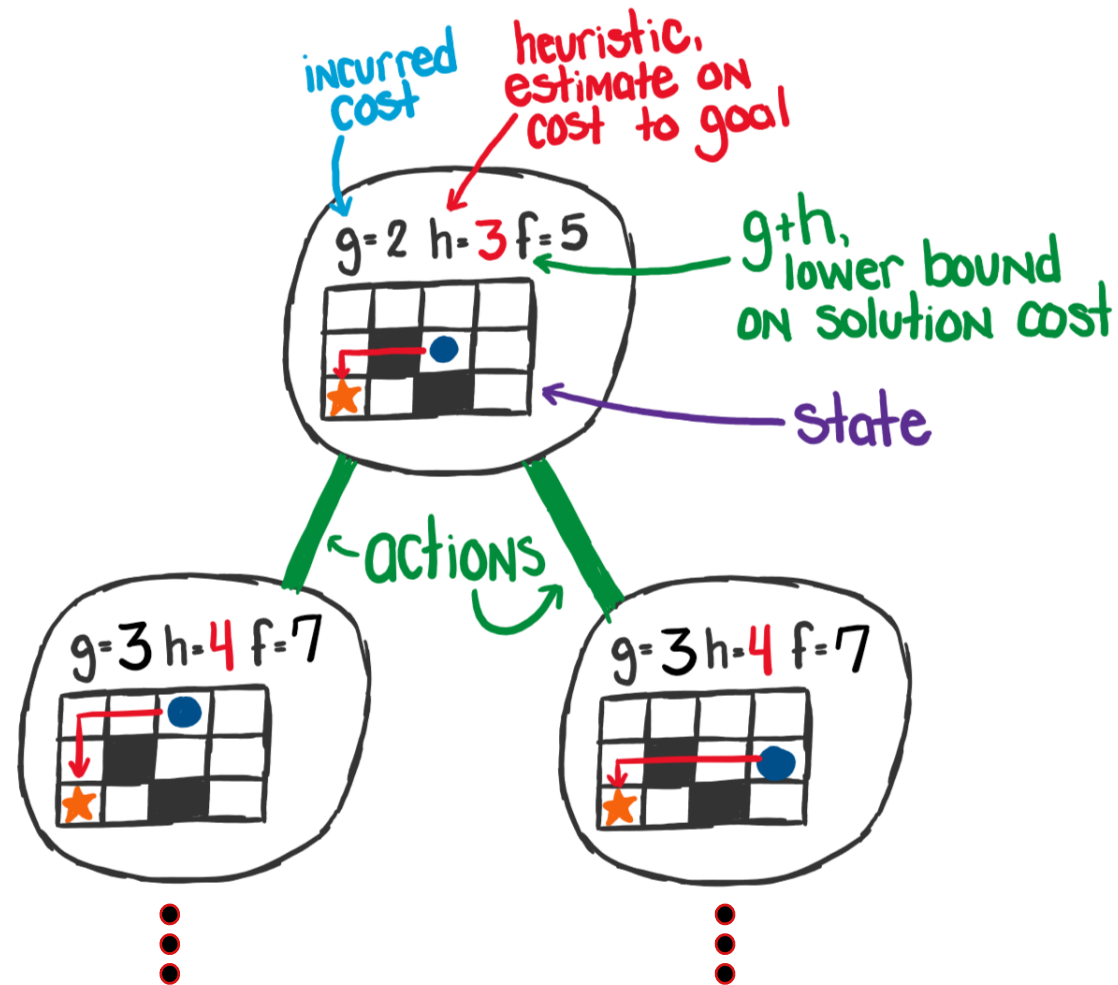
$\{\text{states, actions}\} \rightarrow \{V, E\}$

planning problem  $\rightarrow$  find a path from  $s_{init}$  to  $\{s_{goal}\}$

guide graph search by a heuristic estimate of cost-to-goal

# What is Heuristic Search?

heuristic search associates costs with states,  
used to guide search



(Image credit: Andrew Mitchell)

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**A\*:** expands the node with minimal  $f$  value  
returns optimal path  
**optimal search can take too long!**  
because it must expand every node with  $f < C^*$ <sup>1</sup>

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<sup>1</sup>How Good is Almost Perfect, Malte Helmert and Gabriele Roger, AAAI, 2008.



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What if we don't have time?

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<sup>1</sup>How Good is Almost Perfect, Malte Helmert and Gabriele Roger, AAAI, 2008.

# What If We Are Under Time Pressure?

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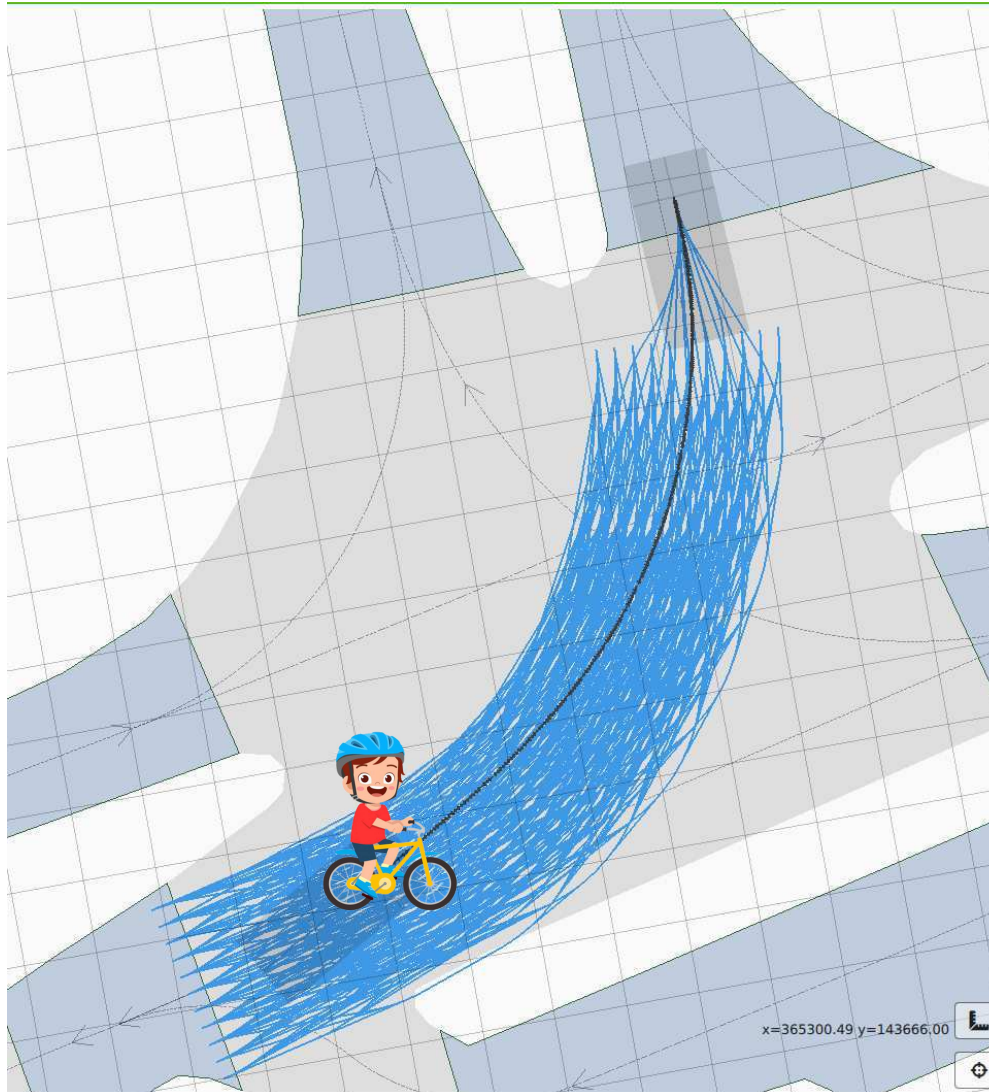
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- large state space
- limited resource
- hard time bound

# Alternatives to Optimal Search?

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1. real-time heuristic search  
time bound
2. concurrent planning and execution  
system must be stay under control
3. bounded-cost search  
cost bound

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metareasoning:

reasoning about which reasoning to do

# Metareasoning: Reasoning About Which Reasoning To Do

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planning

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metareasoning

# Metareasoning: Reasoning About Which Reasoning To Do

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Make sure we are not dying from overthink and never act!



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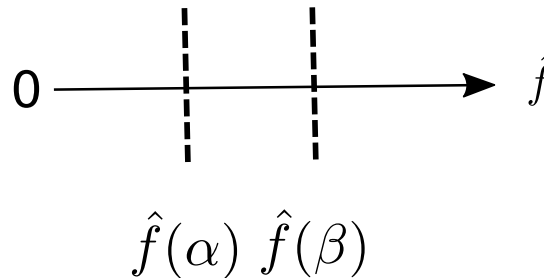
## Online Planning

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bounded rationality  $\rightarrow$  uncertainty

Intuitively, distribution can be better than scalar-value based methods because it quantifies uncertainty, which is what search resolves.



$\hat{f}$  is expected value

Should an agent expand nodes under  $\alpha$  or  $\beta$ ?

# Why Use Distributional Methods?

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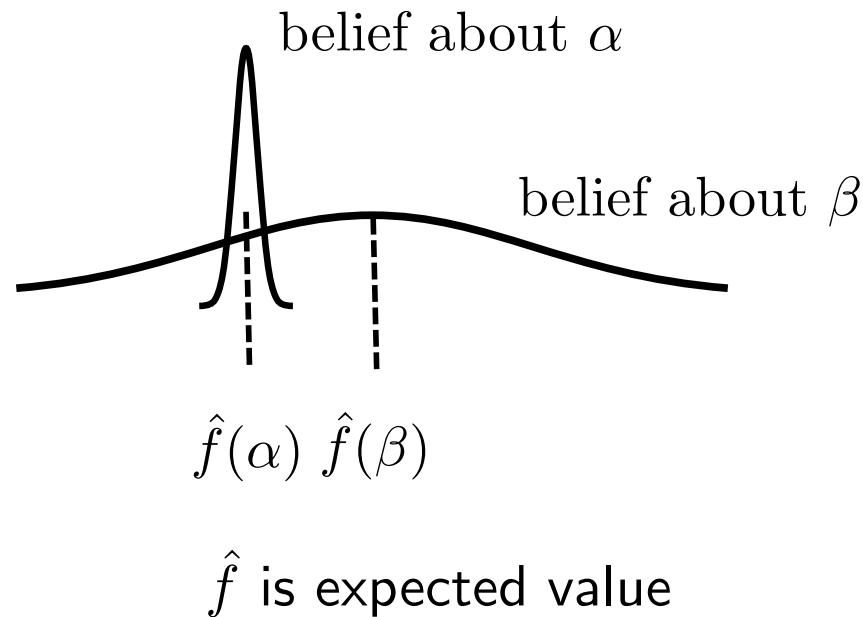
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## heuristic search can benefit from representing uncertainty

- improving real-time search by representing uncertainty  
DDNancy: AAAI-20 \*
- improving concurrent planing and execution by representing uncertainty  
FACS: IntEx-21
- improving bounded-cost search by representing uncertainty  
XES: IJCAI-21

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# Data-driven Real-time Search as Decision-making Under Uncertainty: Data-driven Nancy

**Joint work with Maximilian Fickert, Leonhard Staut,  
Sai Lekyang, Wheeler Ruml, Joerg Hoffmann, and Marek Petrik**

# What is Real-time Heuristic Search?

An example: path finding



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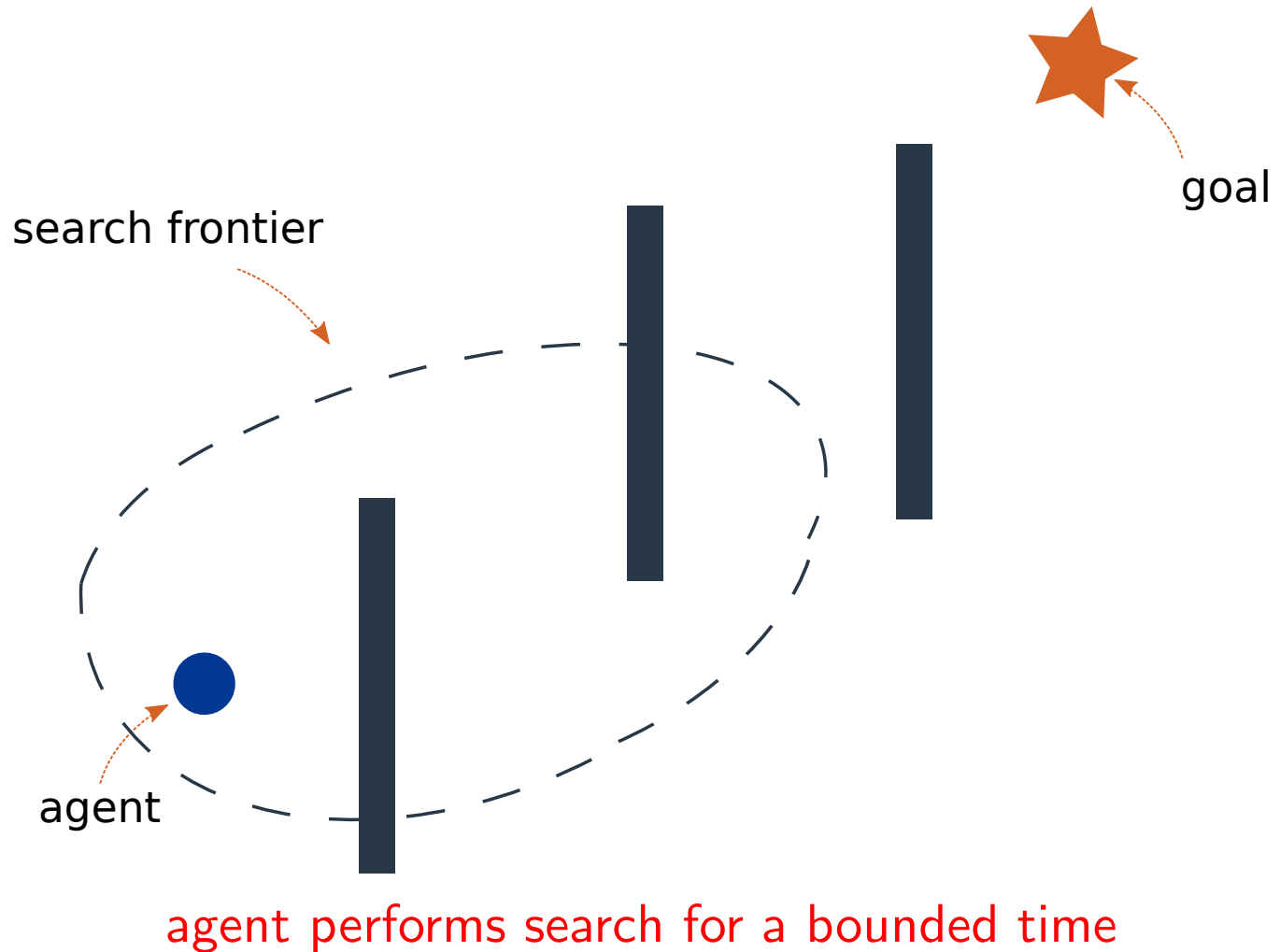
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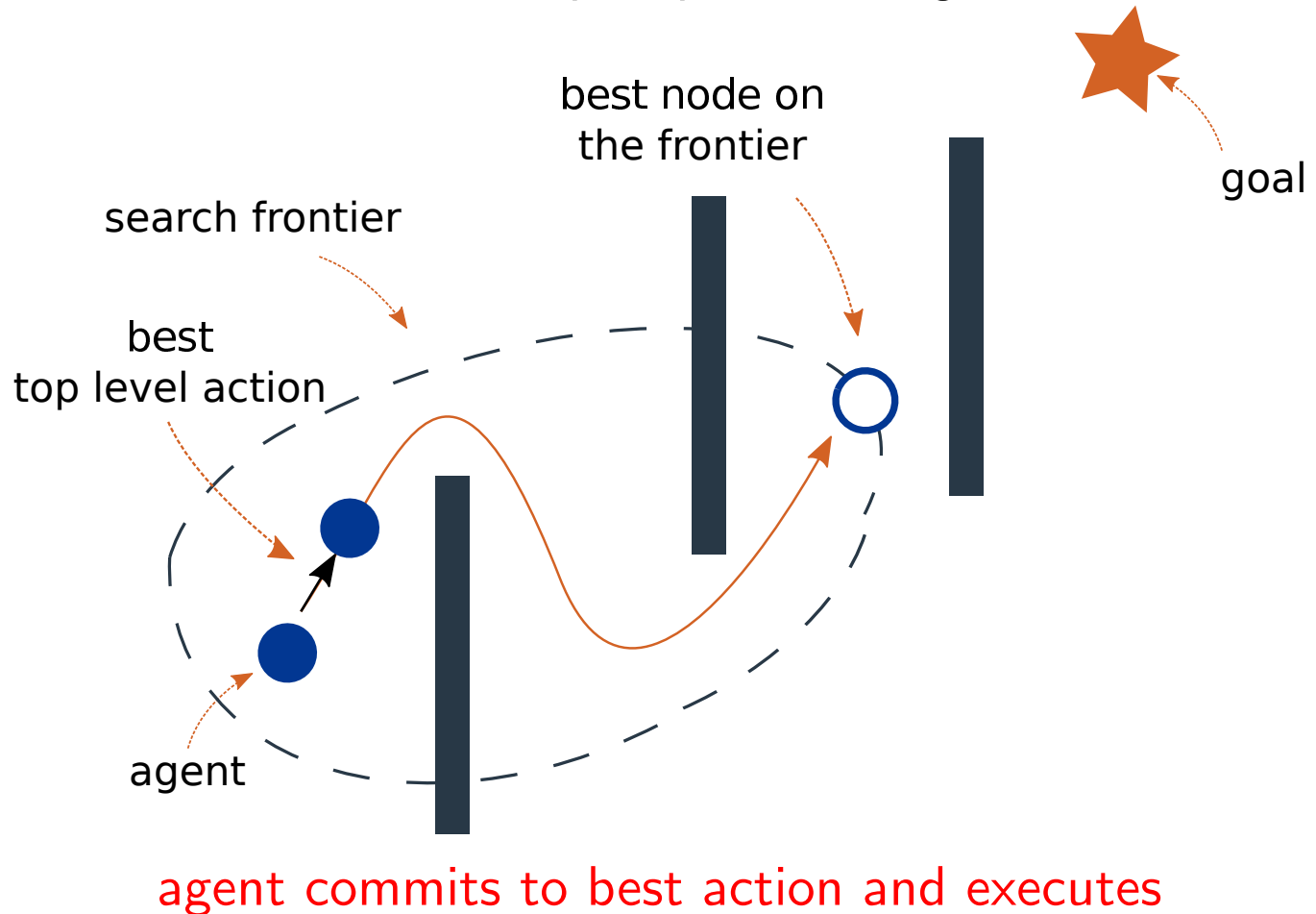
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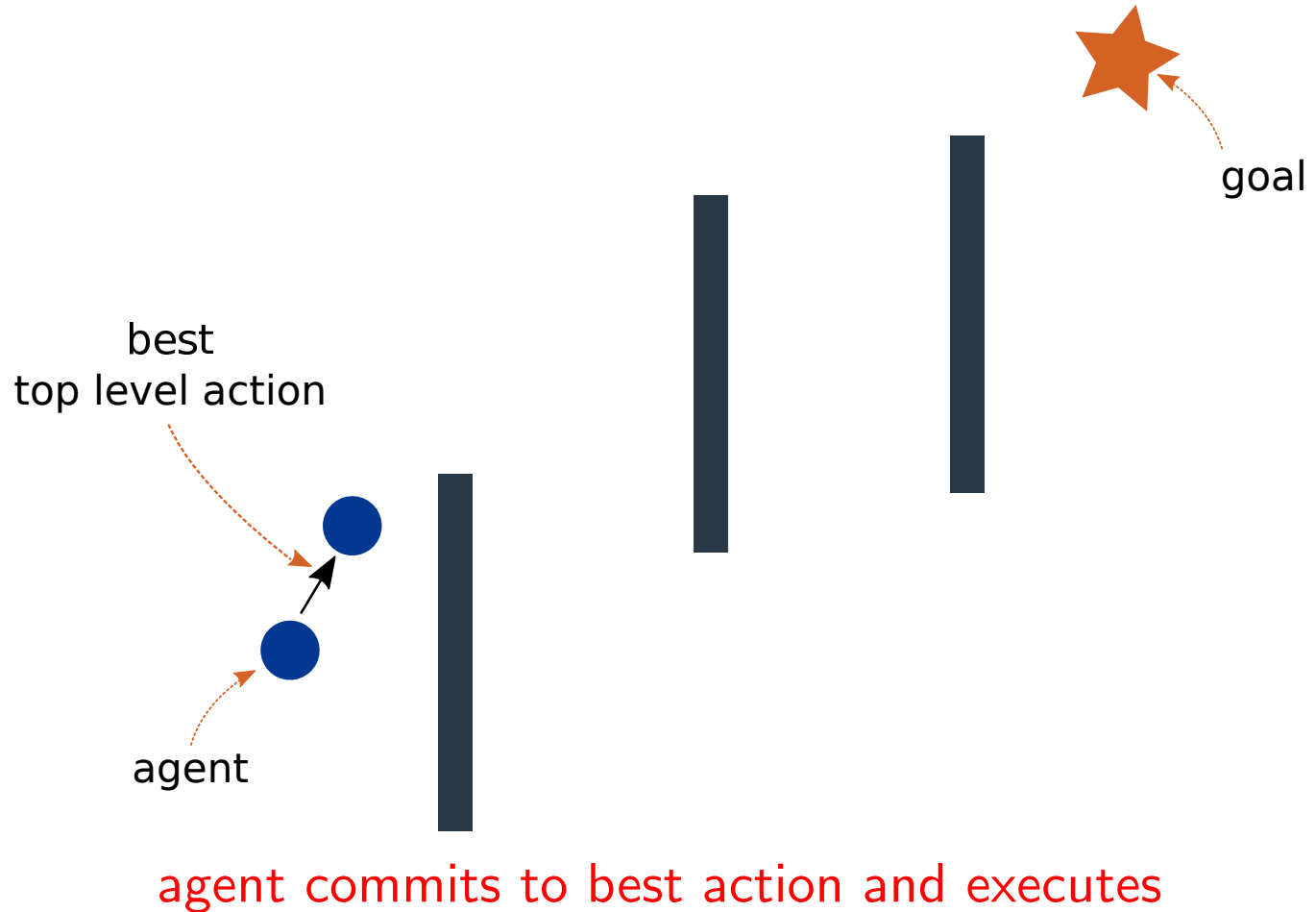
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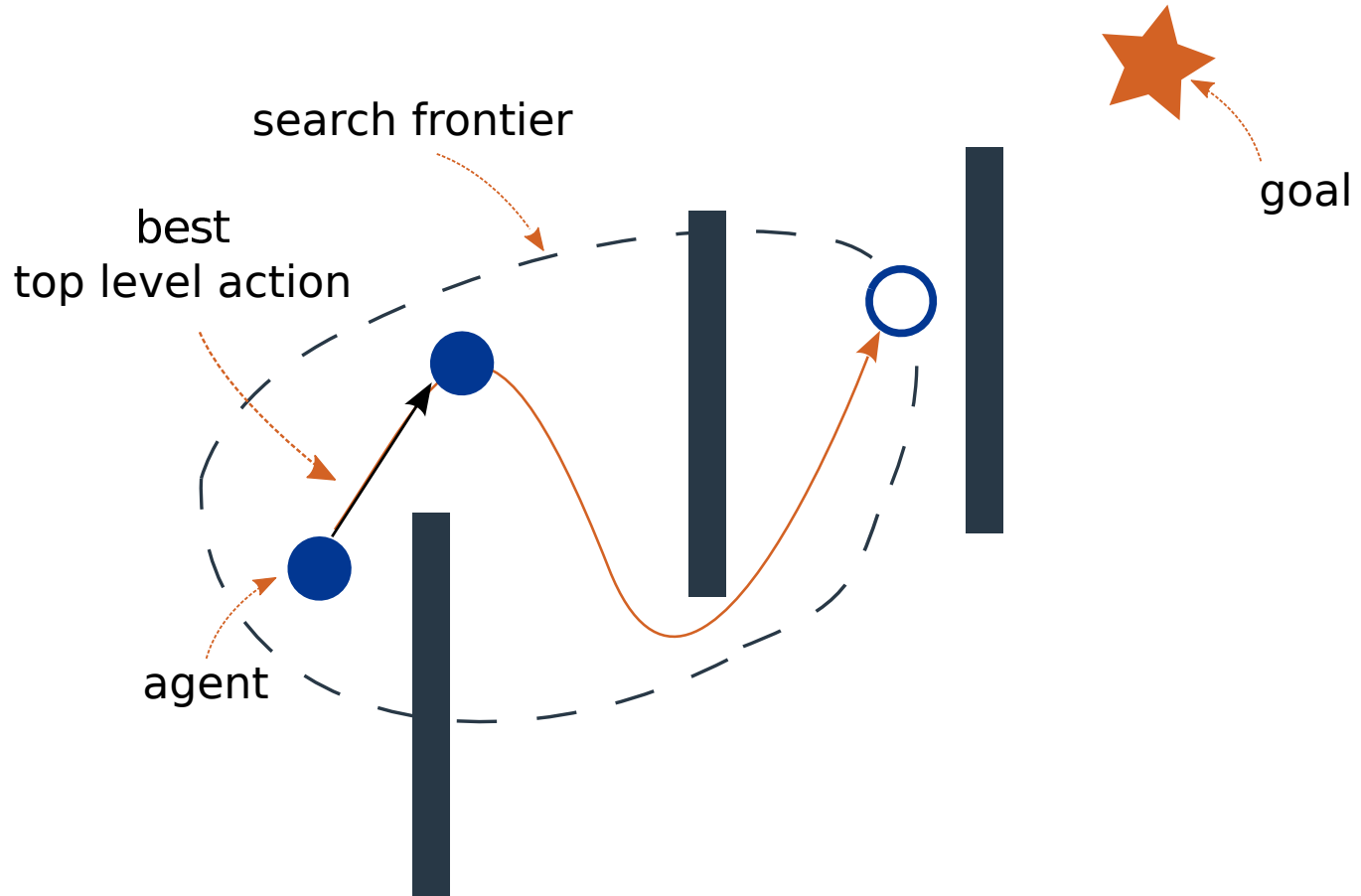
An example: path finding



agent commits to best action and executes

# What is Real-time Heuristic Search?

An example: path finding



online planning: interleaving search and action execution  
“receding horizon control”

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# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

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three phases:

1. Lookahead Phase:  
expands nodes with minimum  $f$   
to explore the search space

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three phases:

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backup the minimum  $f$  from search frontier ('minimin')  
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proved to be complete for consistent heuristic

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proved to be complete for consistent heuristic

derived from offline search, but optimal for online?

# Lookahead Phase: A Troublesome Example

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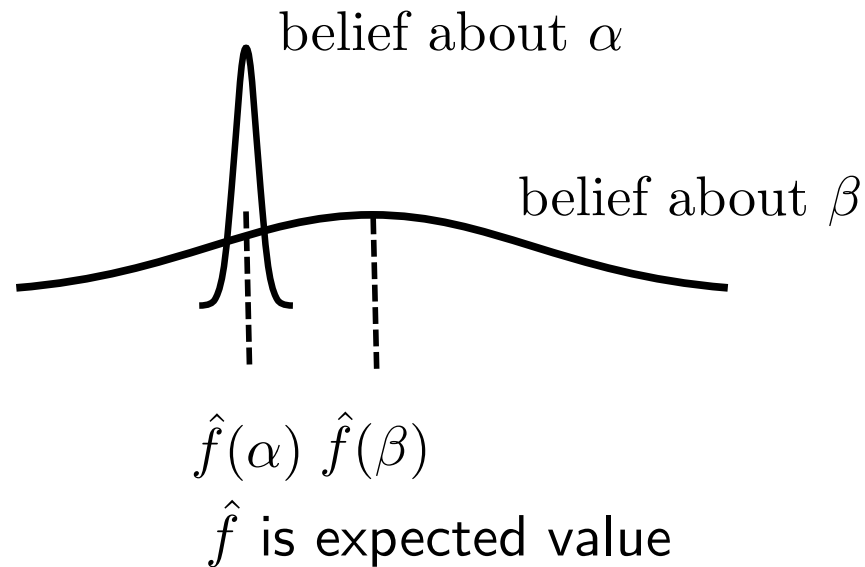
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Should an agent expand nodes under  $\alpha$  or  $\beta$ ?



# Lookahead Phase: A Troublesome Example

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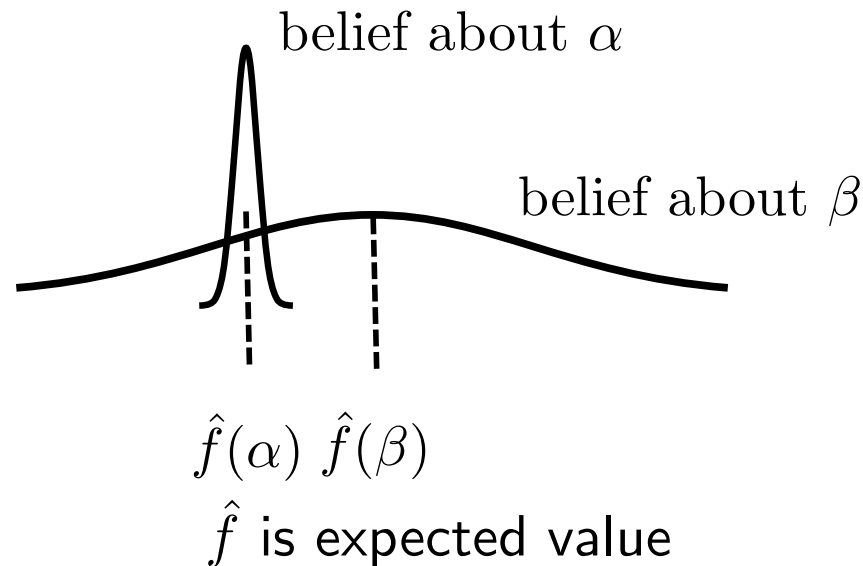
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Should an agent expand nodes under  $\alpha$  or  $\beta$ ?

$\hat{f}$  is not the answer: what to do?

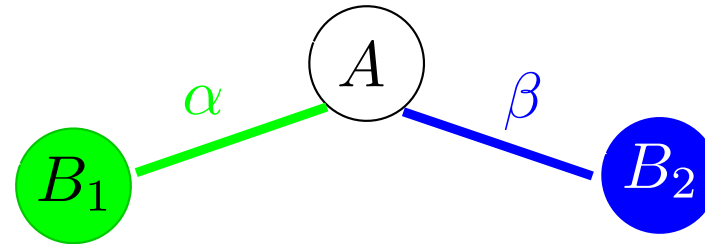
want to maximize value of information

**need to consider uncertainty of estimates**

Risk-based evaluation: minimize expected regret

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?



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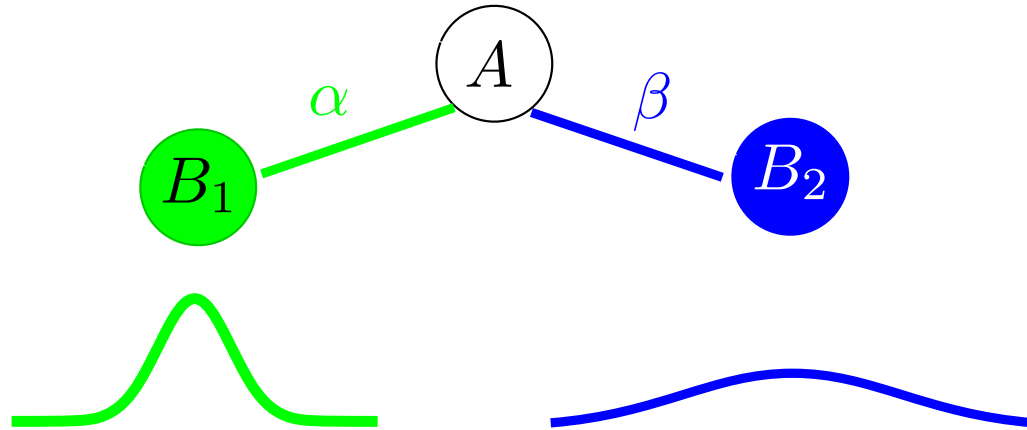
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# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?



need 2 things:

- 1) current beliefs
- 2) estimate of how beliefs might change with search

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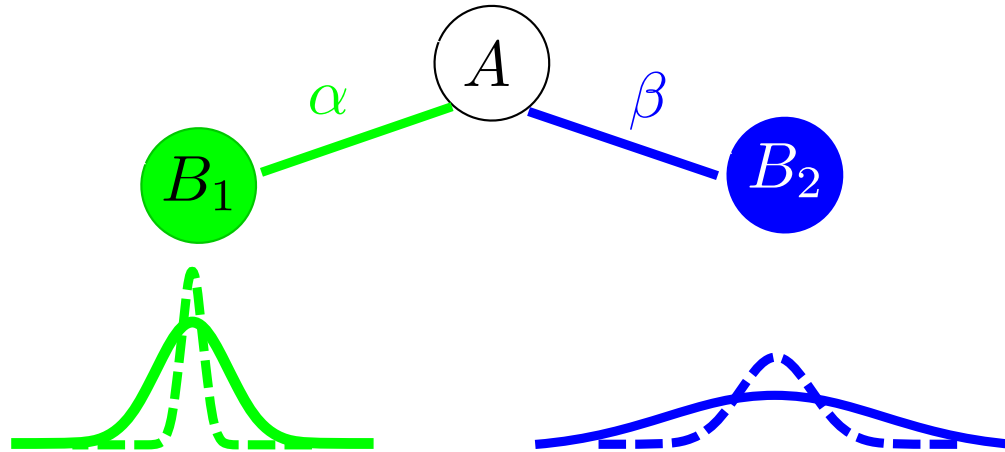
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# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

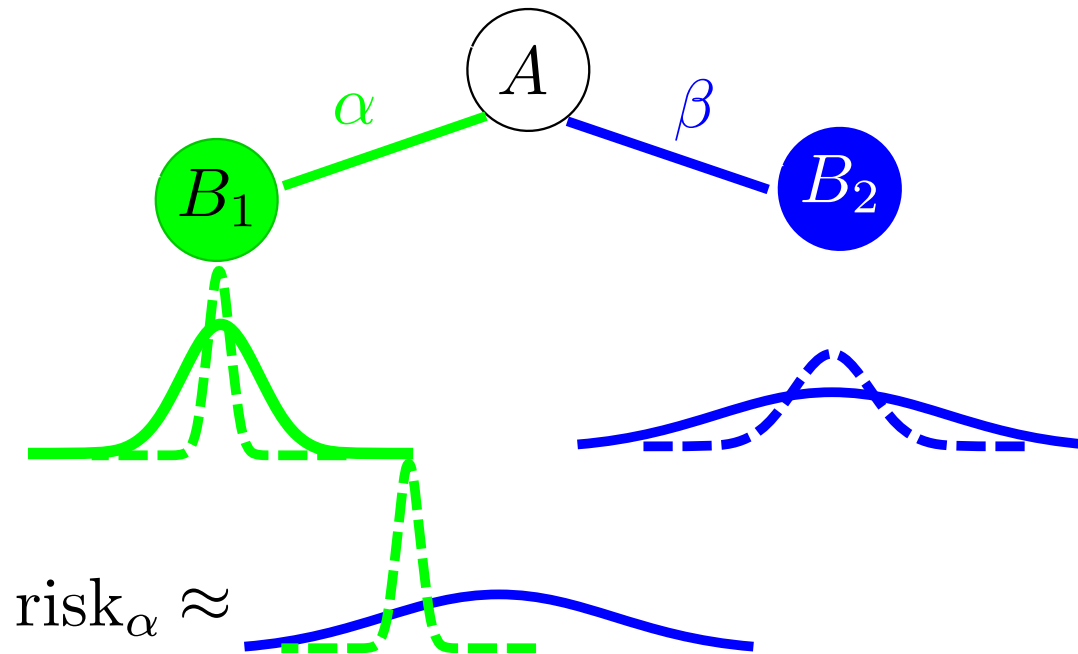


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# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

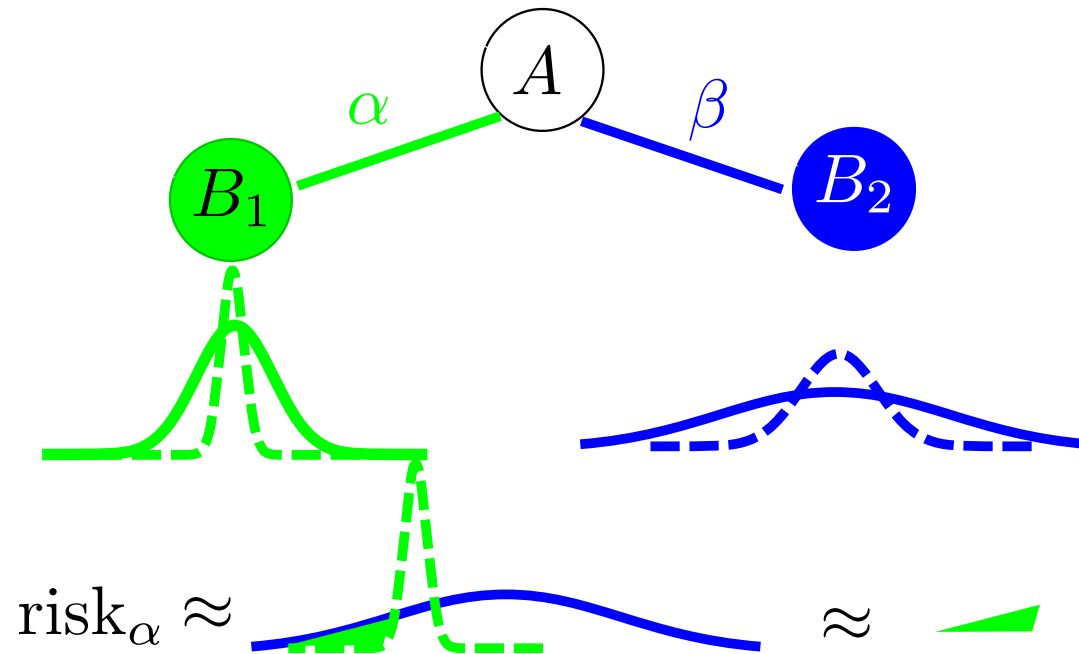


**Risk:** expected regret if a suboptimal action is selected  
 $\alpha$  is TLA with lowest expected value, other is  $\beta$

$$\mathbb{E} \left[ \underbrace{f^*(\alpha) - f^*(\beta)}_{\text{what is our regret}} \mid \underbrace{f^*(\beta) < f^*(\alpha)}_{\text{in cases when } \alpha \text{ not best}} \right]$$

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

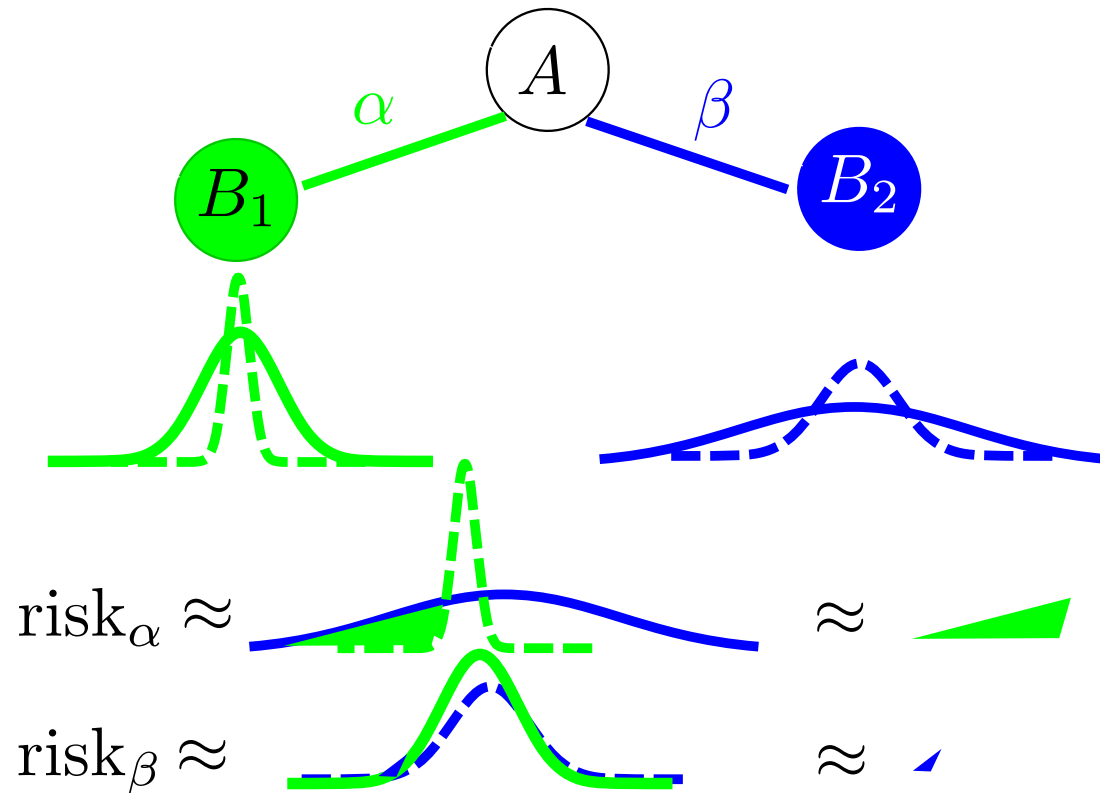


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# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?



expand under the TLA that minimizes risk!  
expand under  $\beta$ !

# How to Form The Belief Distribution?

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Heuristic values: scalar  $\rightarrow$  probability distribution (belief)

But where do beliefs come from?



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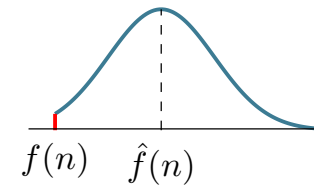
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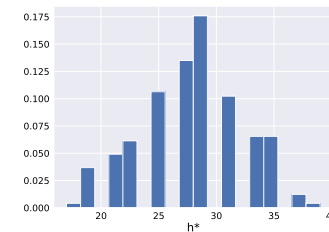
Nancy (Mitchell et al 2019):

truncated Gaussian based on  $\hat{f}$  and  $f$ ,  
few parameters allows **online learning**



My work: Data-Driven Nancy:

expressive histogram,  
many parameters requires **offline learning**



# Learning a Model of Heuristic Error

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belief: distribution of  $h^*$  given features of state ( $h$ )

Gathering data:

run weighted-A\* on random problems and collect all states  
for each observed  $h$  value:

pick most common 200 states from the collection,  
compute  $h^*$

# Learning a Model of Heuristic Error

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run weighted-A\* on random problems and collect all states  
for each observed  $h$  value:

pick most common 200 states from the collection,  
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compute  $h^*$ : need powerful optimal solver (eg. IDA\*<sub>CR</sub><sup>2</sup> with  
pattern database heuristic)

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<sup>2</sup>Reducing reexpansions in iterative-deepening search by controlling cutoff bounds, U.K. Sarkar, P.P. Chakrabarti, S. Ghose, S.C. De Sarkar, Artificial Intelligence, 1991.

# Example $h^*$ distribution: Sliding Puzzle

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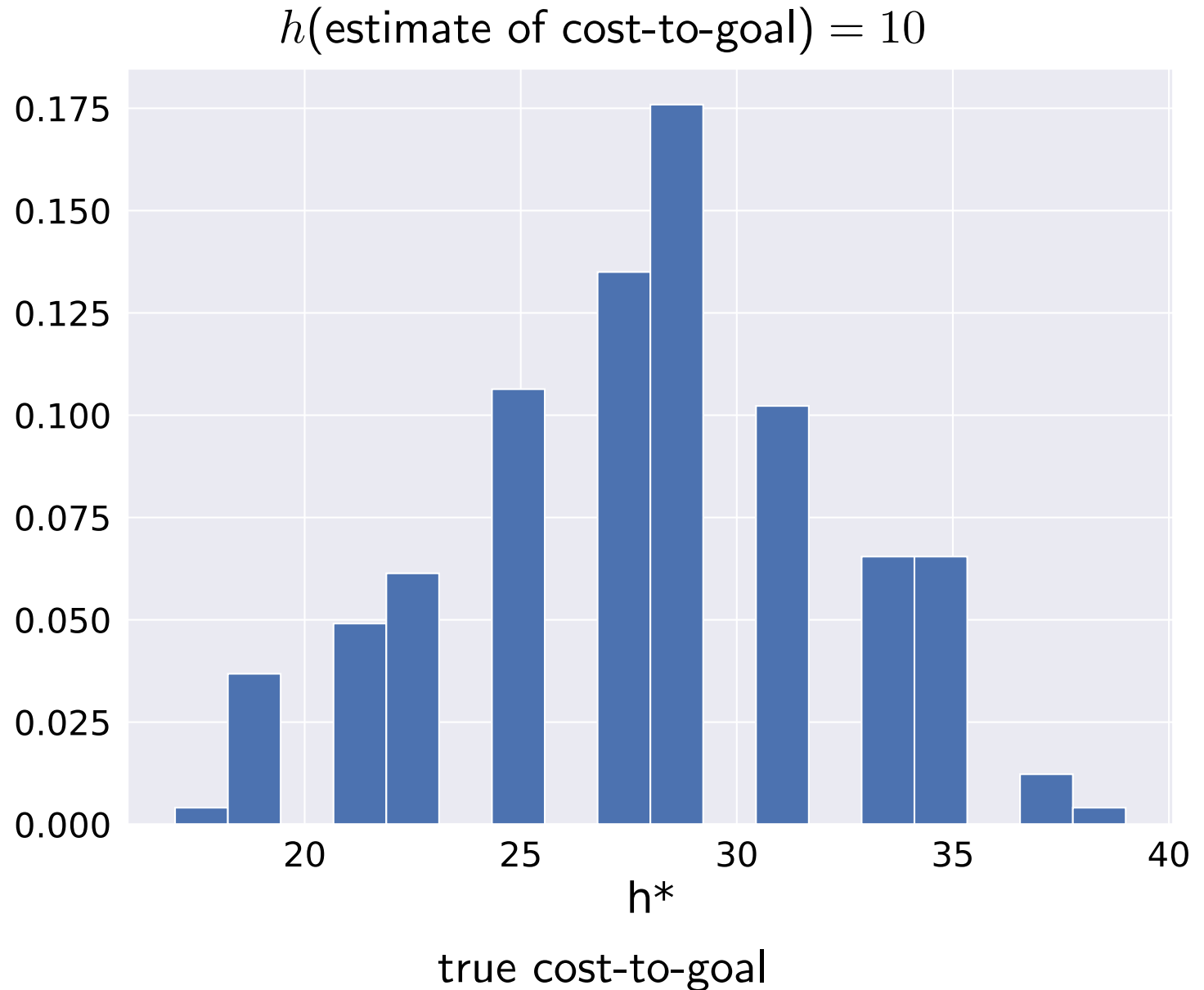
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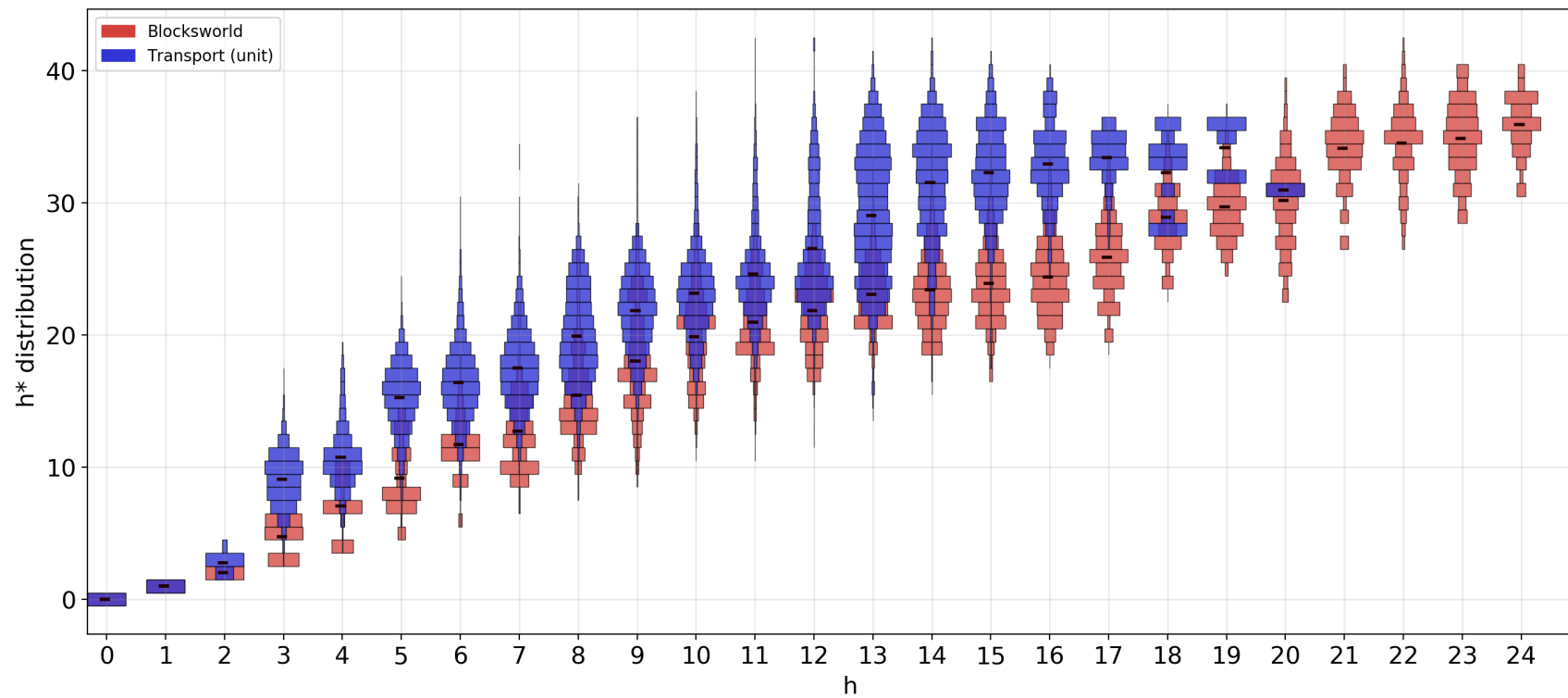
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# Example $h^*$ distribution: Transport vs Blocks World

What does the actual cost-to-go value uncertainty distribution look like?



Beliefs are different from domain to domain  
(Image credit: Leonhard Staut)

# Results on Heavy Sliding Puzzle Problem

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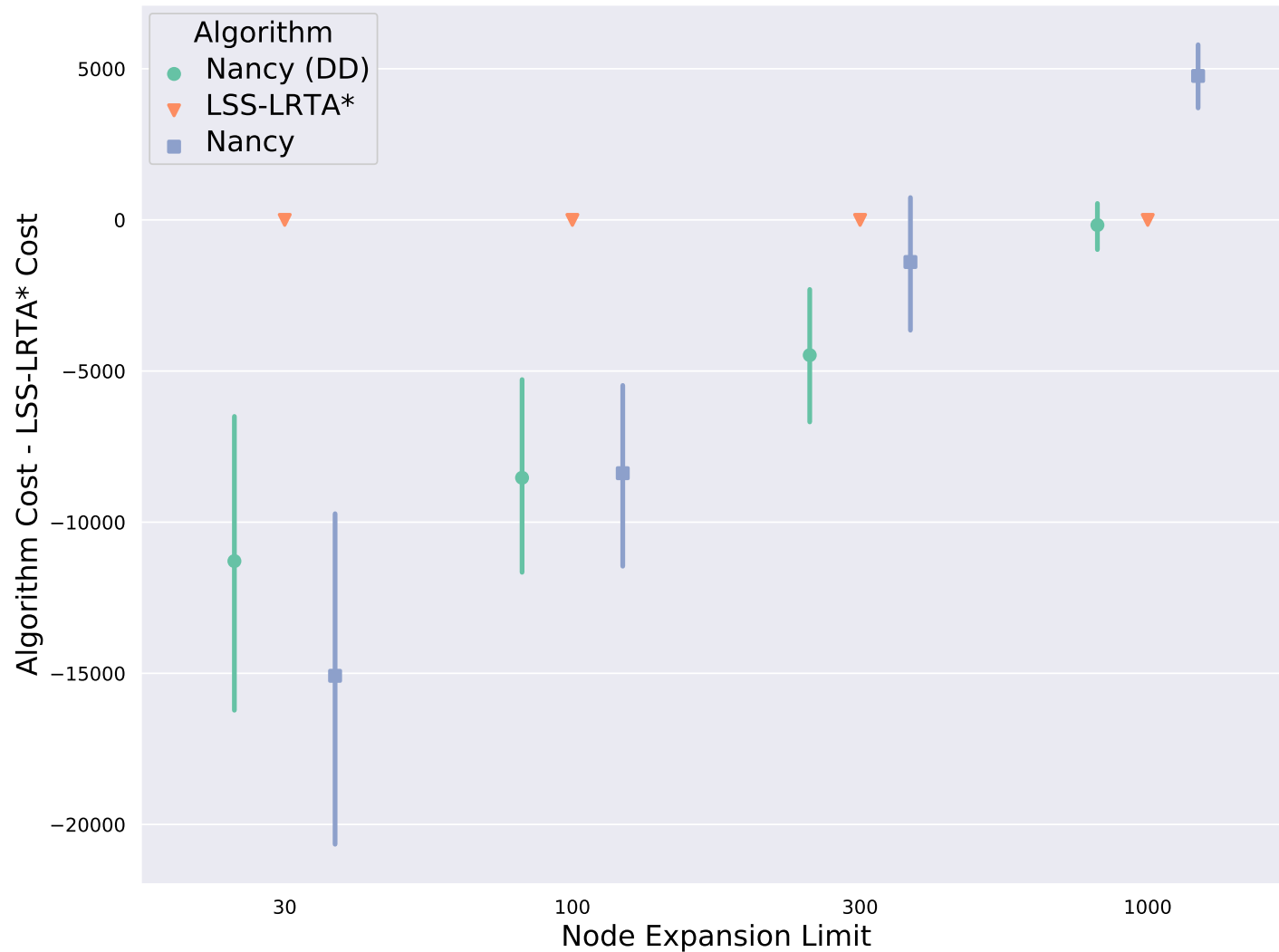
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Both version of Nancy outperform conventional approach!

# Results on Racetrack Problem

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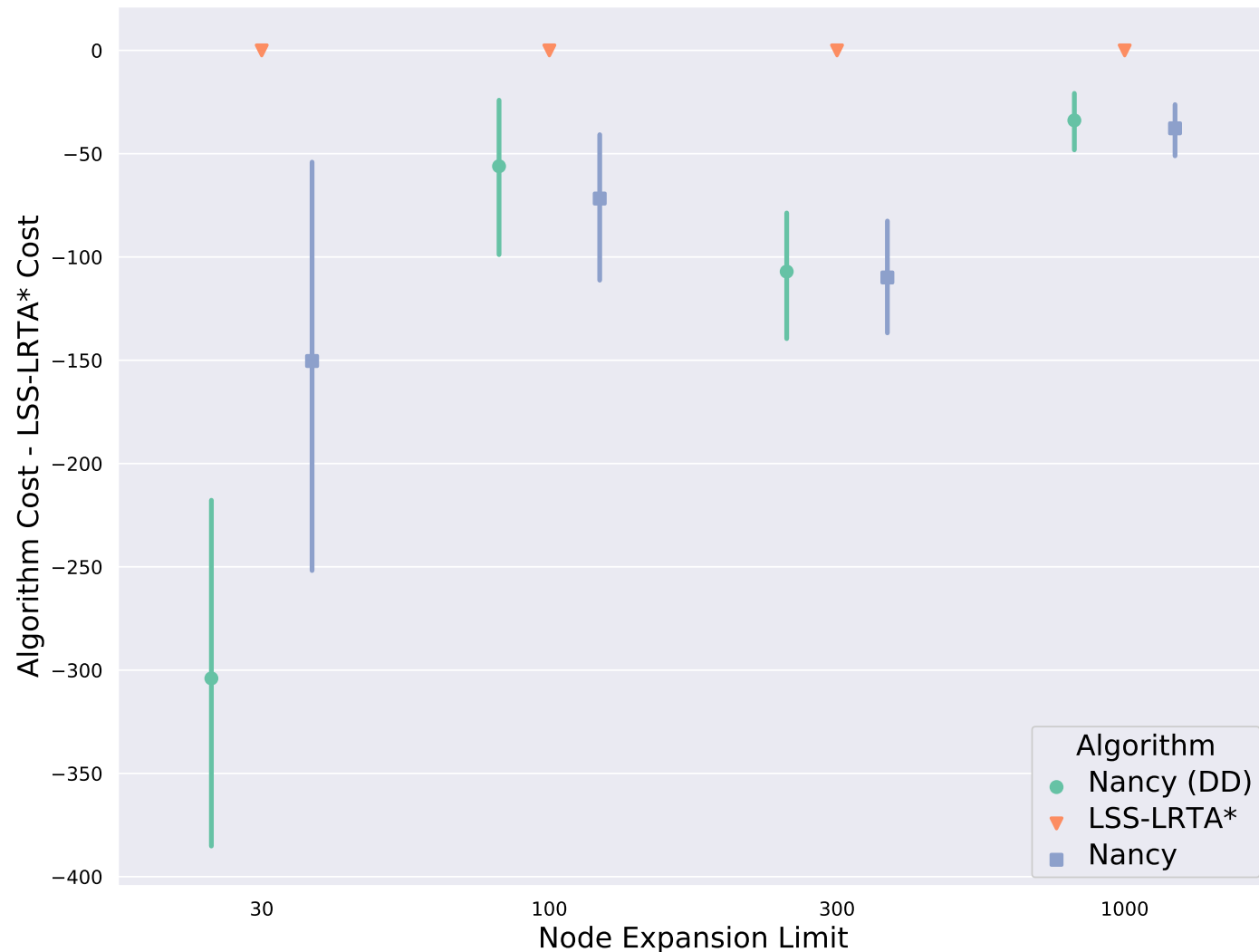
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Data-Driven Nancy is more robust!

# Mean Solution Cost on Planning Domains

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Domain	Lookahead	LSS- LRTA*	Nancy	Nancy (DD)
Blocksw.	100	46	<b>33</b>	38
	300	36	<b>30</b>	34
	1000	30	32	<b>27</b>
Transport	100	631	615	<b>496</b>
	300	519	559	<b>485</b>
	1000	499	567	<b>422</b>
Transport (unit-cost)	100	48	40	<b>31</b>
	300	47	<b>30</b>	34
	1000	35	29	<b>27</b>
Elevators (unit-cost)	100	50	<b>35</b>	39
	300	32	<b>29</b>	30
	1000	34	27	<b>26</b>

**Data-Driven Nancy is more robust!**

(Table credit: Maximilian Fickert, Leonhard Staut)



# CPU Time on Sliding Puzzle Problem

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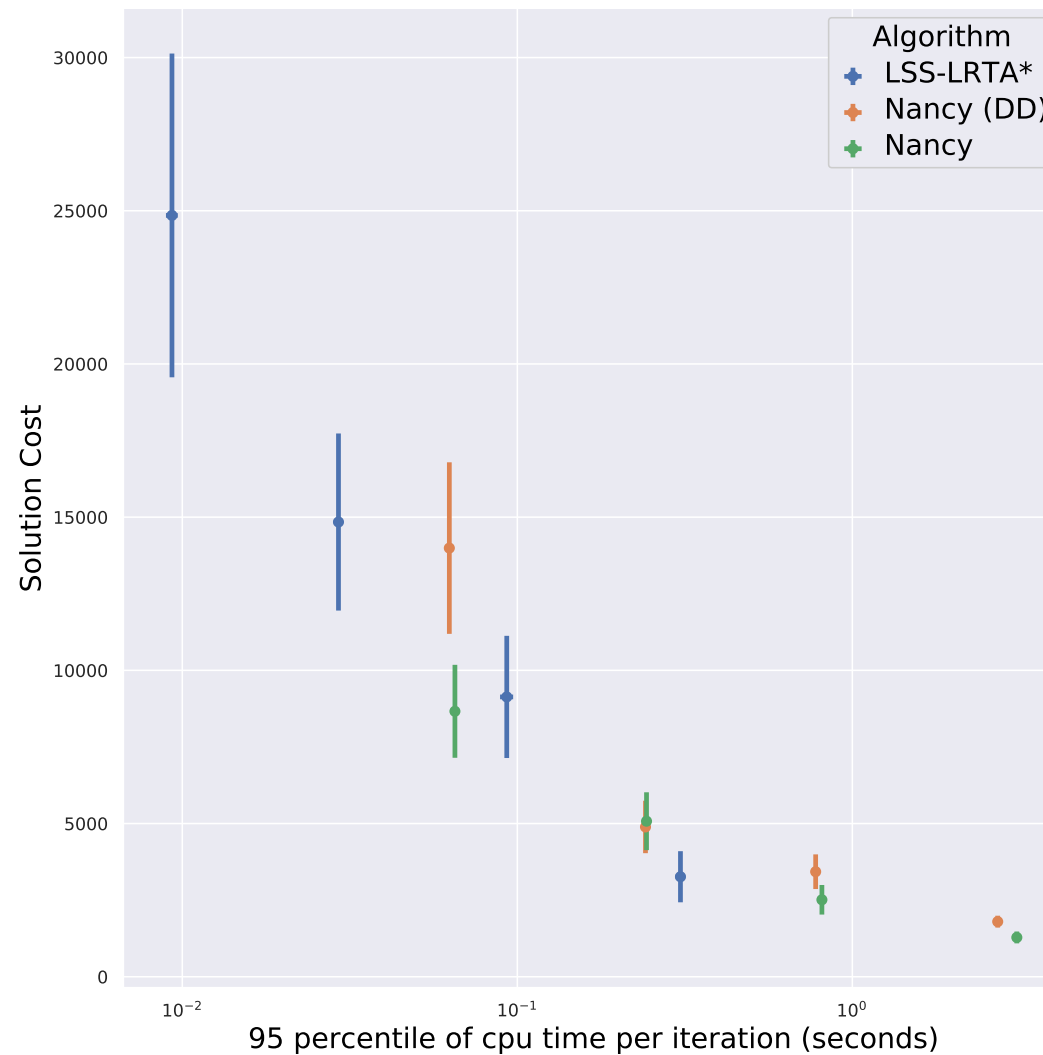
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Nancy incurs overhead but worth it!

# Comparison to IE and MCTS

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Interval Estimation:

IE chooses the TLA with the lowest lower bound on the 95% confidence interval of the backed-up belief

# Comparison to IE and MCTS

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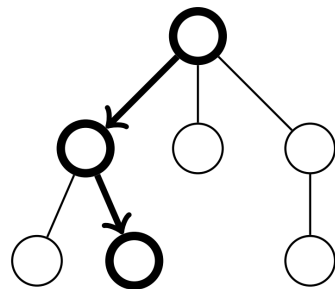
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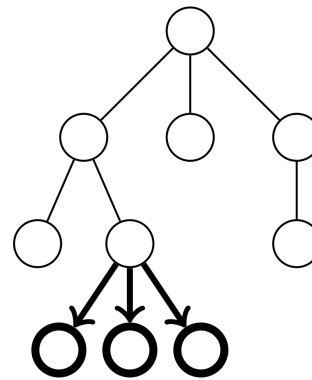
IE chooses the TLA with the lowest lower bound on the 95% confidence interval of the backed-up belief

MCTS

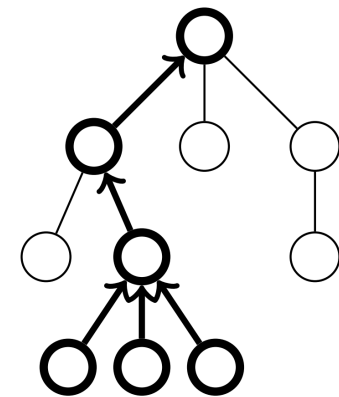
use THTS-WA\* in the expansion phrase



Action Selection



Initialization



Backup function

(Image credit: Tim Schulte and Thomas Keller)

# Comparison to IE and MCTS on 40 Pancake

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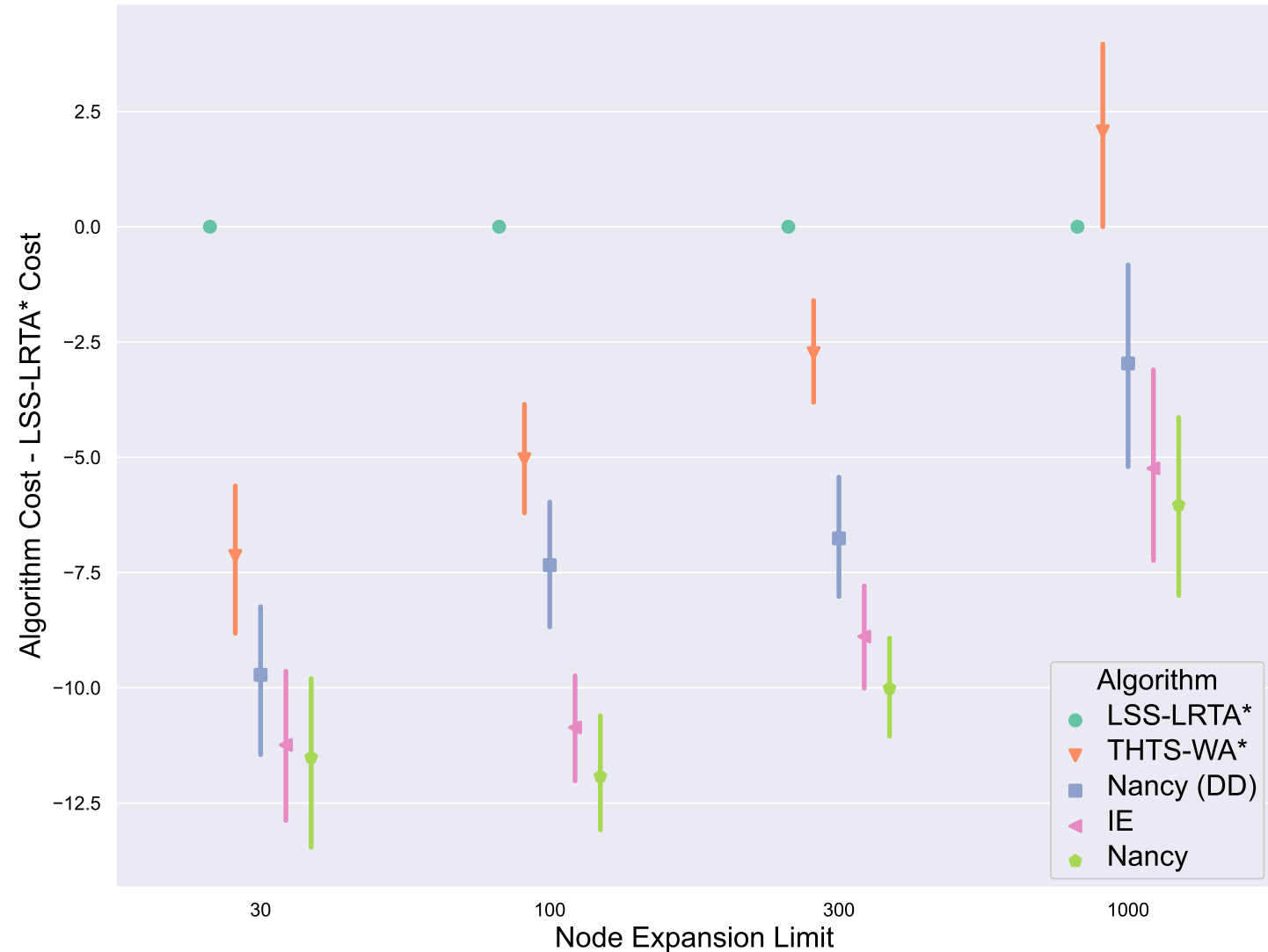
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Reasoning about uncertainty helps!

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- Distributional methods provide significant improvements compared to conventional LSS-LRTA\* in real-time search
- Nancy starts to explore an optimal way of doing online heuristic search
- Data-driven approach provides an alternative way to implement the Nancy framework, it performs better when assumption fails

## Status of the work:

1. *Beliefs We Can Believe In: Replacing Assumptions with Data in Real-Time Search*. AAAI Conference on Artificial Intelligence (AAAI), 2020.
2. *Real-time Planning as Data-driven Decision-making*. ICAPS Workshop on Bridging the Gap Between AI Planning and Reinforcement Learning (PRL), 2020.
3. In preparation: *Real-time Planning as Decision-making Under Heuristic Value Uncertainty*, Journal of Artificial Intelligence Research (JAIR)

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# When to Commit to an Action in Online Planning?

**Joint work with Shahaf Shperberg, Eyal Shlomo Shimony,  
Wheeler Ruml and Erez Karpas**

# An Example of Online Planning Using Heuristic Search

An example: highway navigation

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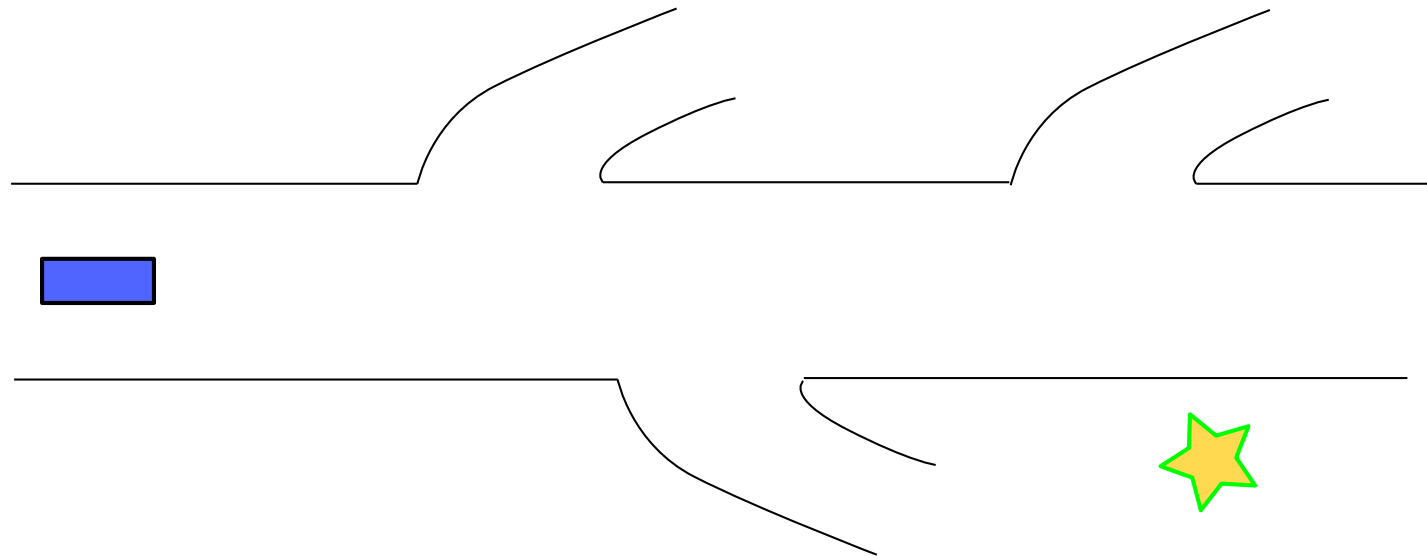
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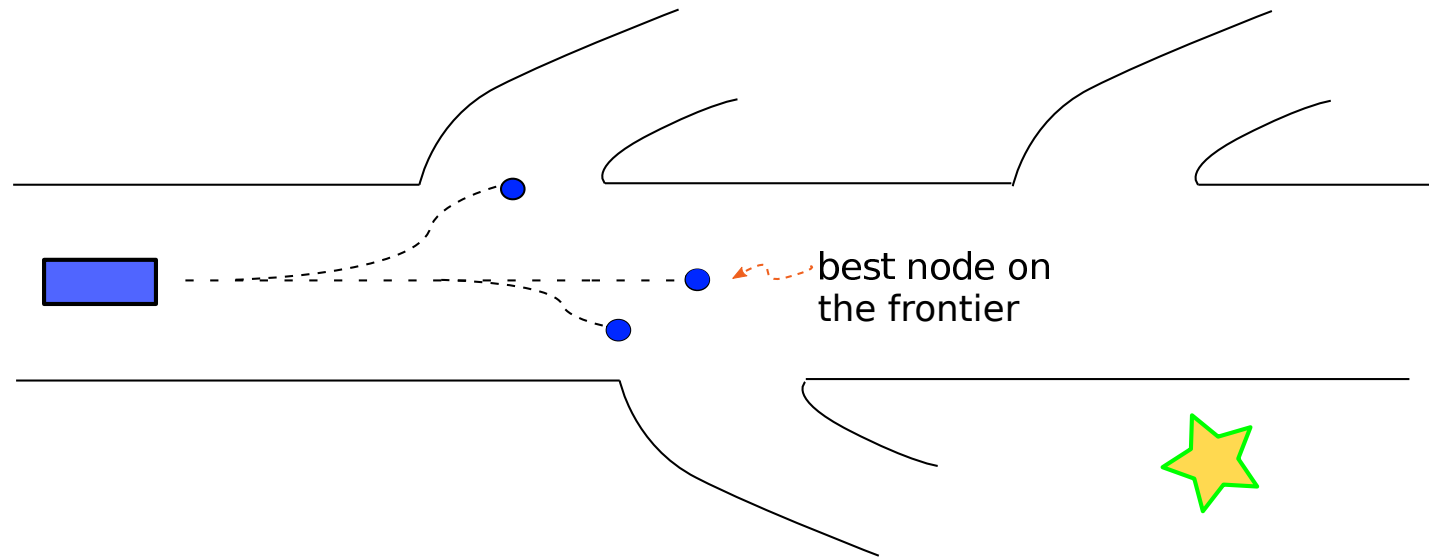
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agent performs search for a bounded time

# An Example of Online Planning Using Heuristic Search

## An example: highway navigation



agent performs search for a bounded time

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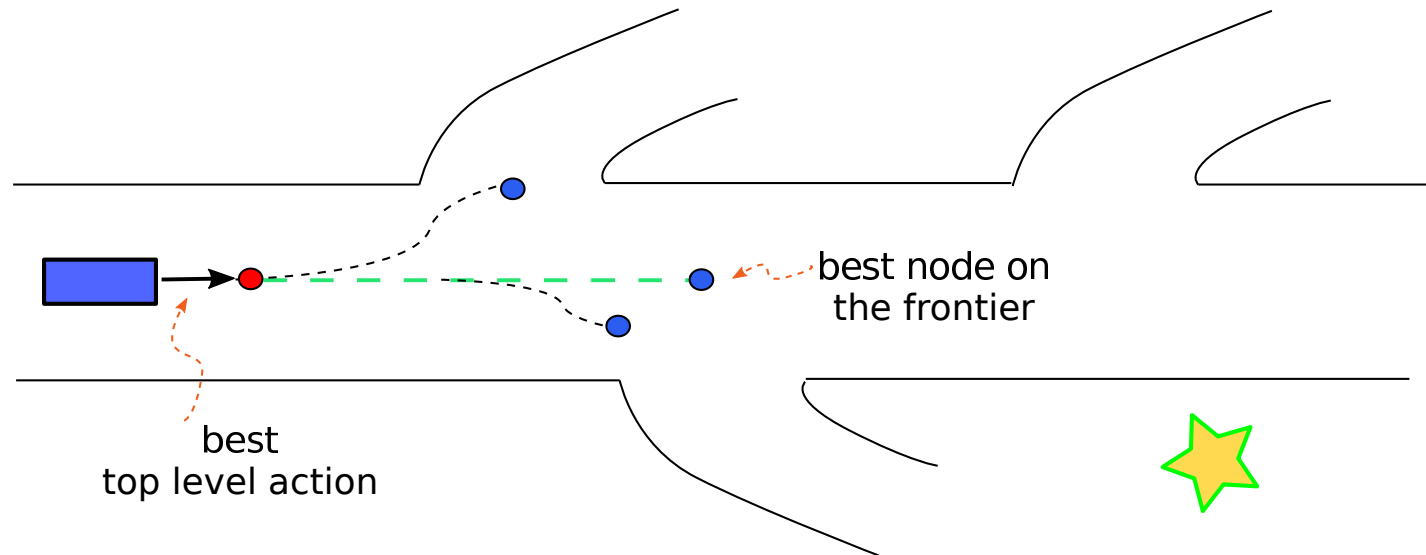
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# An Example of Online Planning Using Heuristic Search

## An example: highway navigation



agent commits to best action and executes

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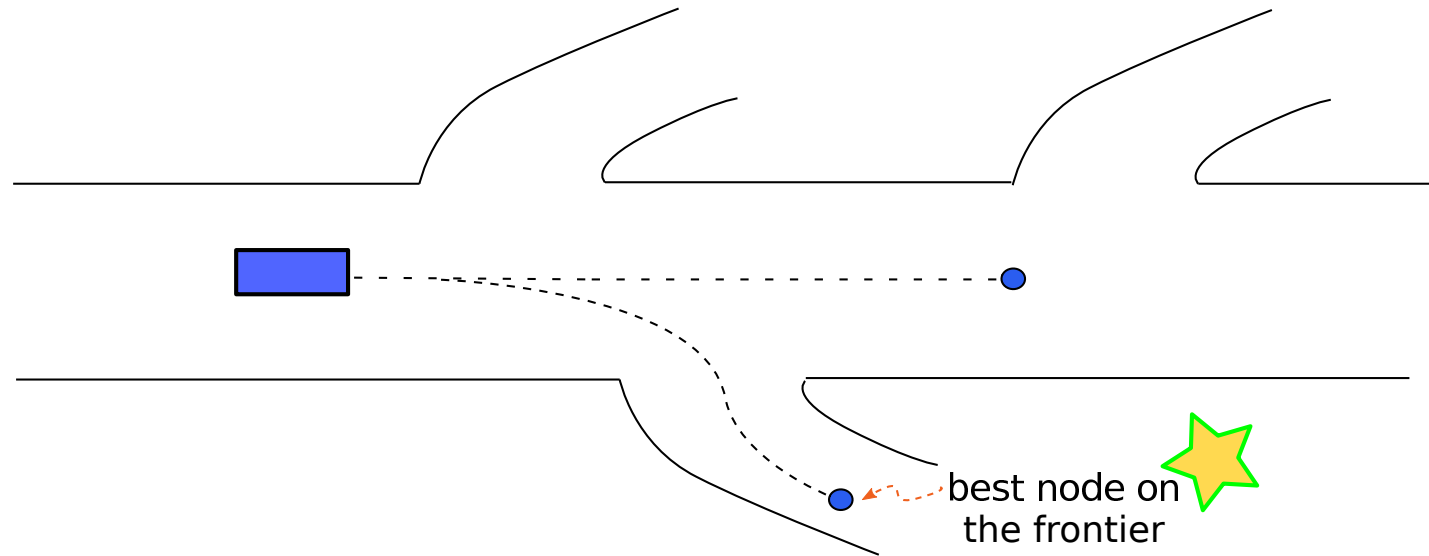
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# An Example of Online Planning Using Heuristic Search

An example: highway navigation



agent commits to best action and executes

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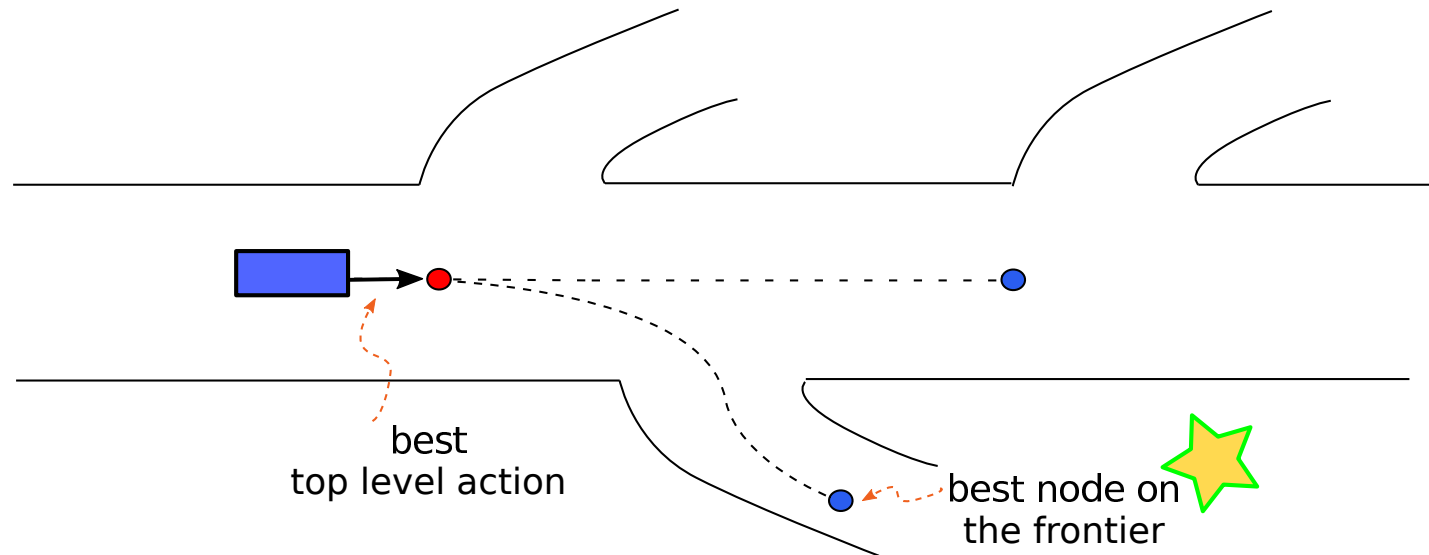
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# An Example of Online Planning Using Heuristic Search

An example: highway navigation



online planning: interleaving search and action execution  
“receding horizon control”

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# The Meta-level Problem: Commit or Not Commit

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For each node along the best prefix path:  
should we commit?

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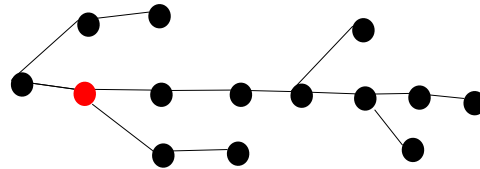
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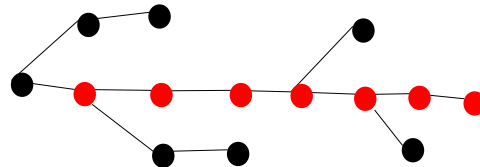
For each node along the best prefix path:  
should we commit?

fixed strategies:

always commit one (Korf 1990)



always commit all (Koenig&Sun 2008, Burns et al 2013)



Can we do better?

# The Meta-level Problem: Commit or Not Commit

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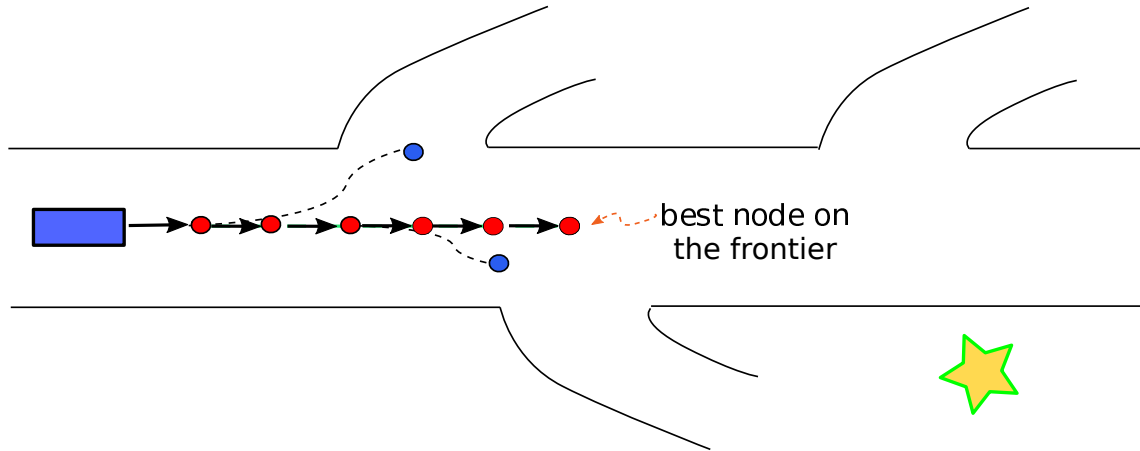
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always commit all is **too risky**



# The Meta-level Problem: Commit or Not Commit

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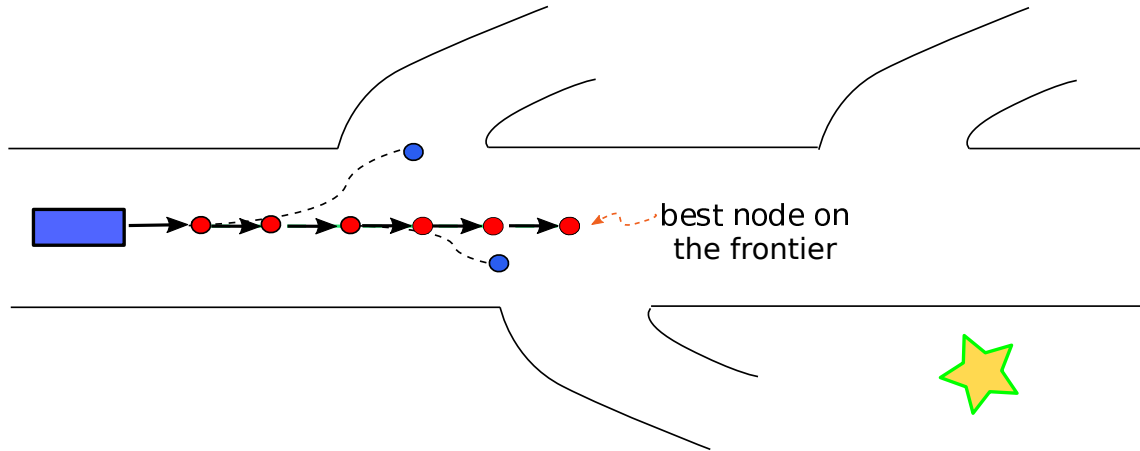
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always commit all is **too risky**



always commit one is **too conservative**

# The Meta-level Problem: Commit or Not Commit

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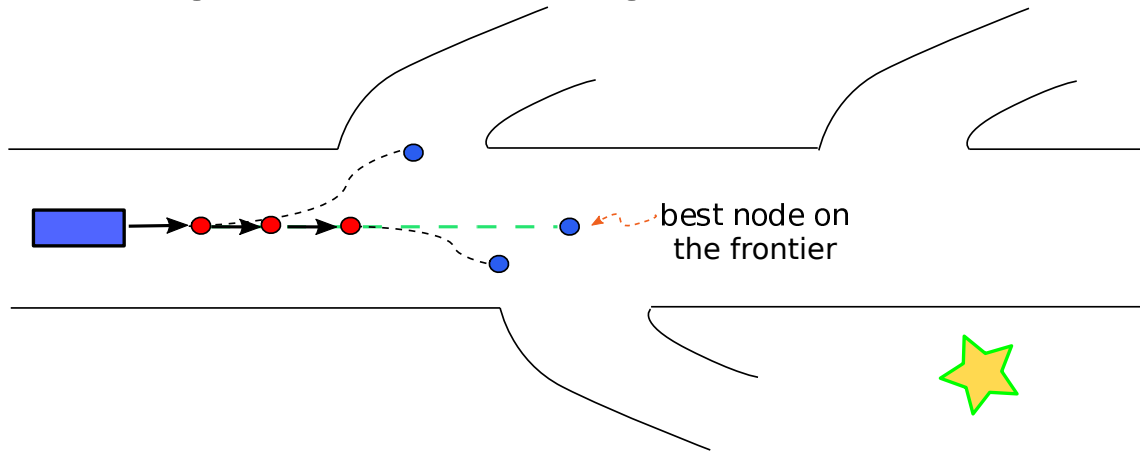
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ideal:

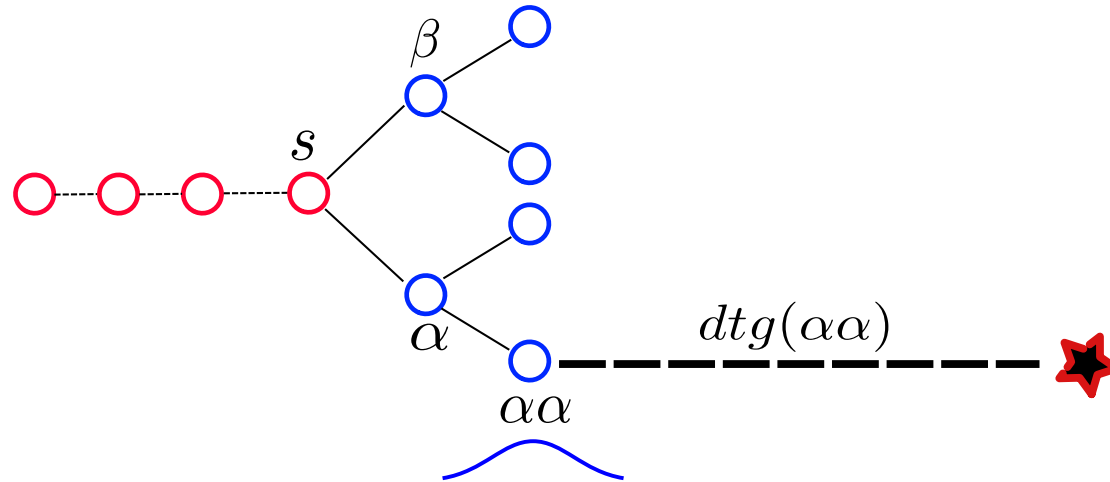
commit if **an action in prefix is certainly the best**  
to gain more planning time for next iteration





# Our Approach: Flexible Action Commitment Search (FACS)

we propose a principled way to make meta-level decision



$$U_{\text{commit}} = \mathbb{E} \left[ \min(X_{\alpha\alpha}^d, X_{\alpha\beta}^d) \right]$$

$$U_{\text{don't commit}} = P_{\text{choose } \alpha} \cdot U_{\alpha} + (1 - P_{\text{choose } \alpha}) \cdot U_{\beta}$$

# Results

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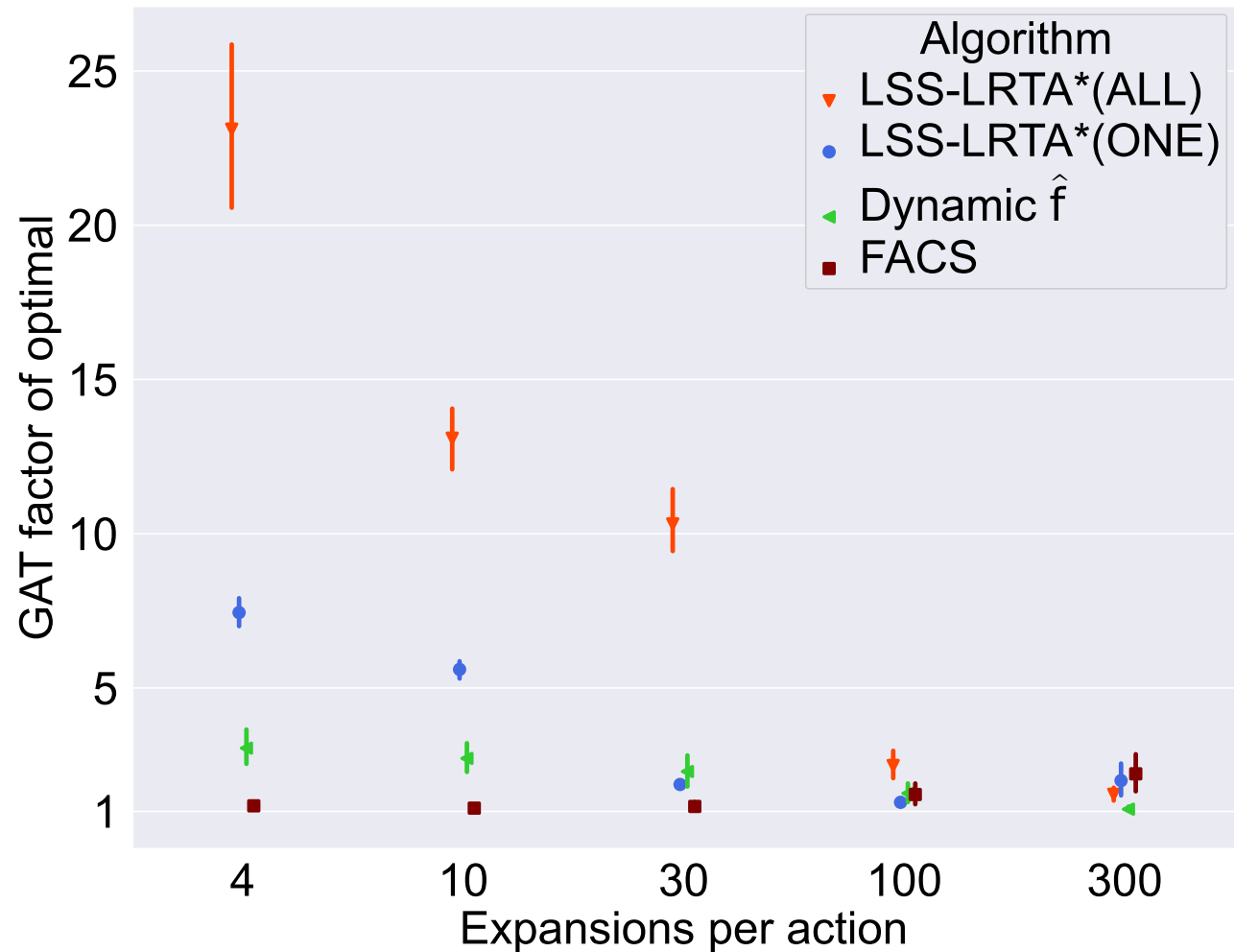
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FACS consistently performs the best

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- FACS starts to explore a principled way of doing online action commitment by reasoning uncertainty
- FACS is better than fixed baseline strategies in synthetic grid pathfinding scenarios.
- Deliberation on how to allocate search effort can benefit online planning

Status of the work:

1. *When to Commit to an Action in Online Planning*. ICAPS Workshop on Integrating, Planning, Acting, and Execution (IntEx-21), 2021
2. In preparation: *When to Commit to an Action in Online Planning*., International Symposium on Combinatorial Search (SoCS 2022)
3. In preparation: *Situated Safe Interval Path Planning for Dynamic Environments*., International Joint Conference on Artificial Intelligence (IJCAI 2022)

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# Bounded-Cost Search Using Estimates of Uncertainty

**Joint work with Maximilian Fickert and Wheeler Ruml**

# What is Bounded-Cost Search?

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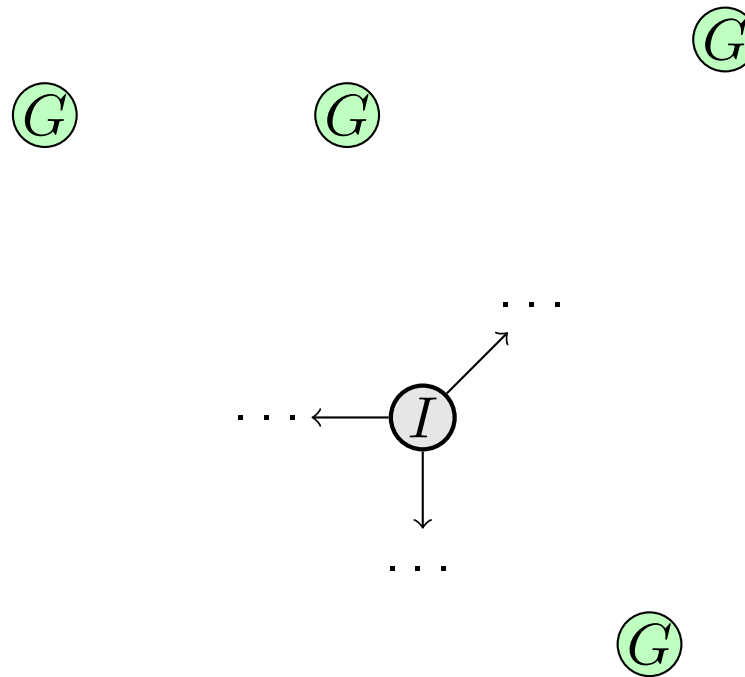
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# What is Bounded-Cost Search?

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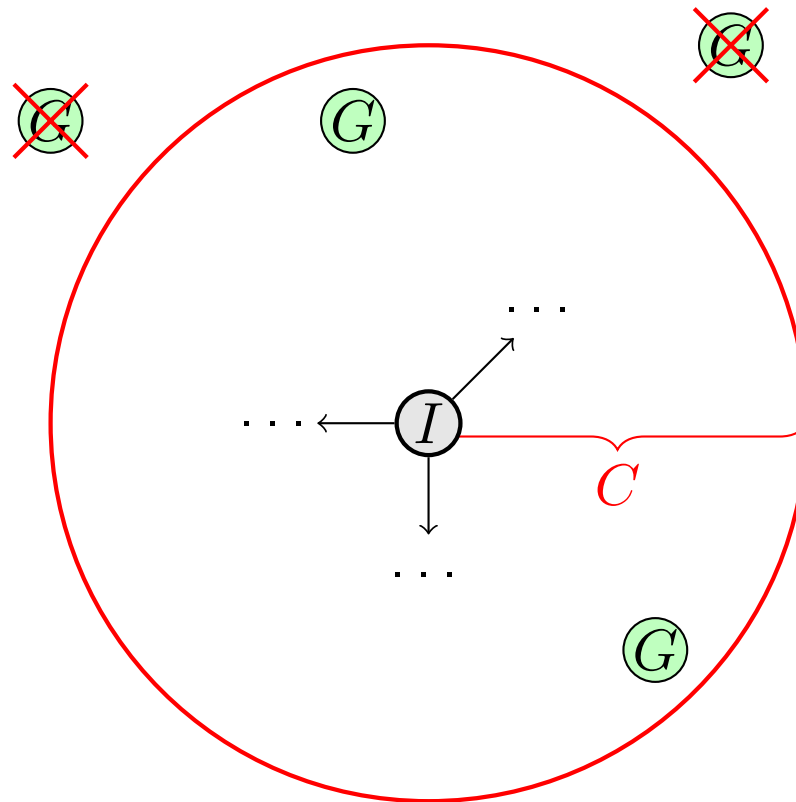
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*Objective: Find a plan with cost at most  $C$  as fast as possible.*

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Potential Search:

best first search on  $f_{l_{nr}}(n) = \frac{h(n)}{C-g(n)}$

does not consider search effort

BEES:

expand the node, among those estimated

to be within the bound, that is closest to a goal

does not consider the uncertainty of its estimate (brittle)

# Our Approach: Expected Effort Search

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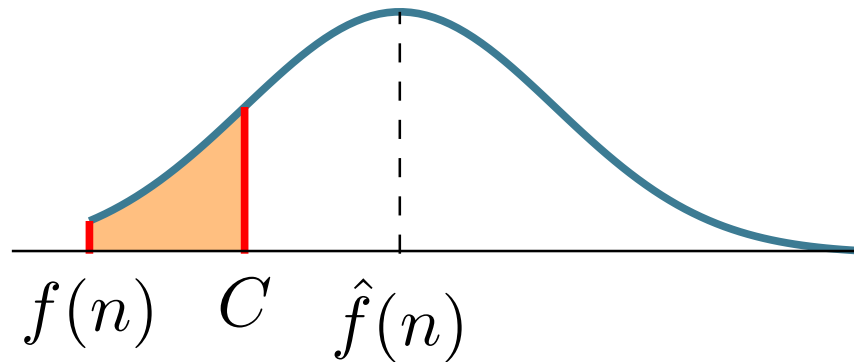
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1. Explicitly estimate the probability of finding a solution within bound  $p(n)$



2. estimate total search effort by  $d(n)$
3. best first search on expected search effort  $d(n)/p(n)$

optimal efficiency proof under three unrealistic assumptions (see dissertation for detail)



# Our Approach: Expected Effort Search

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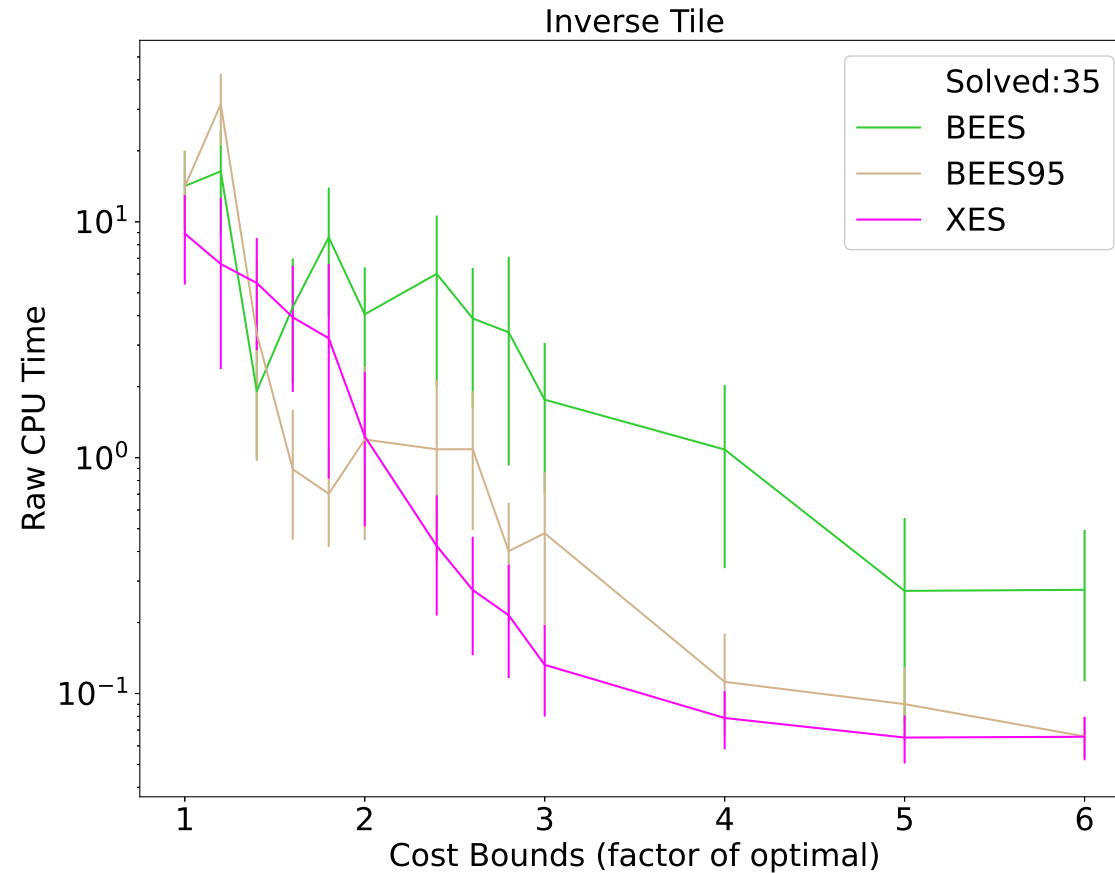
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**XES finds a valid solution faster!**

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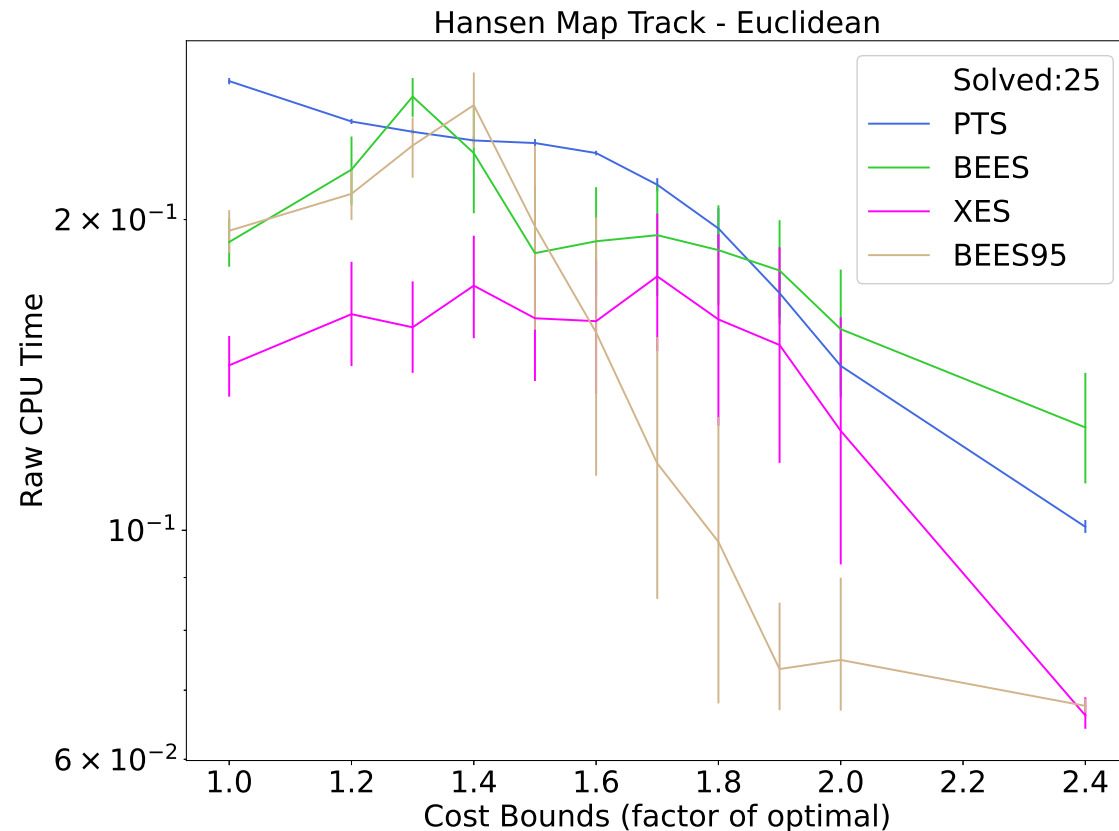
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Status of the work:

1. *Bounded-Cost Search Using Estimates of Uncertainty*. International Joint Conference on Artificial Intelligence (IJCAI), 2021.
2. under review: *New Results in Bounded-Suboptimal Search*. AAAI Conference on Artificial Intelligence (AAAI), 2022.

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# Conclusion

The thesis of my dissertation:

heuristic search can benefit from representing uncertainty

- real-time search
- concurrent planning and execution
- bounded-cost search

# Questions?

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- $f$ -hat
- Expected Effort Search (XES)
- FACS Detail
- FACS Belief
- FACS Decision
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## Back-up Slides

# How To Compute F-Hat

---

$$\hat{f} = g + \hat{h} = g + h + \epsilon d$$

$\hat{f}$  is the expected value for optimal plan cost

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# Expected Effort Search (XES)

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Best-first search on the expected effort:  $\frac{T}{p}$

- $T(n)$ : search effort to find a solution under  $n$
- $p(n)$ : probability that  $n$  leads to a solution within  $C$



# Expected Effort Search (XES)

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Best-first search on the expected effort:  $\frac{T}{p}$

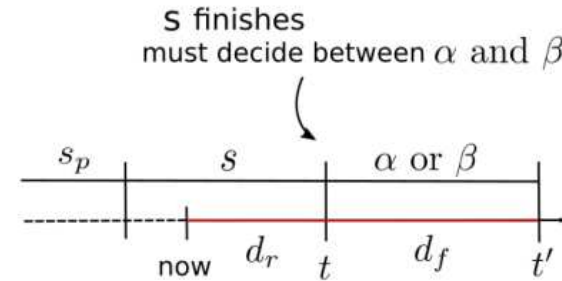
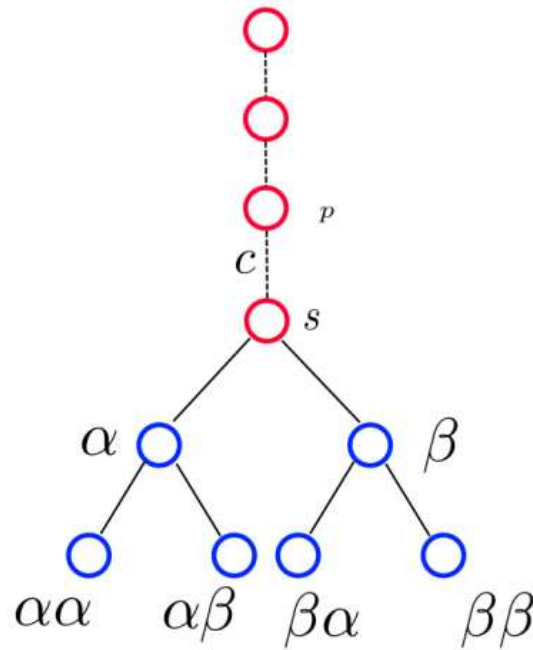
- $T(n)$ : search effort to find a solution under  $n$
- $p(n)$ : probability that  $n$  leads to a solution within  $C$

$$\begin{array}{l} n_1 \\ T = 10 \\ p = 0.5 \end{array} \rightsquigarrow 20$$

$$\begin{array}{l} n_2 \\ T = 6 \\ p = 0.25 \end{array} \rightsquigarrow 24$$

# Our Approach: Flexible Action Commitment Search (FACS)

we propose a principled way to make meta-level decision



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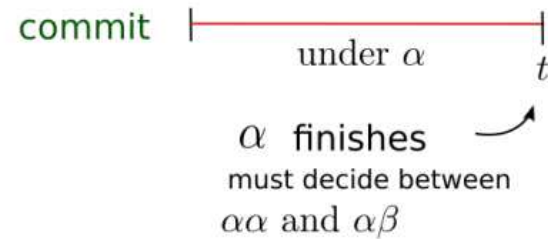
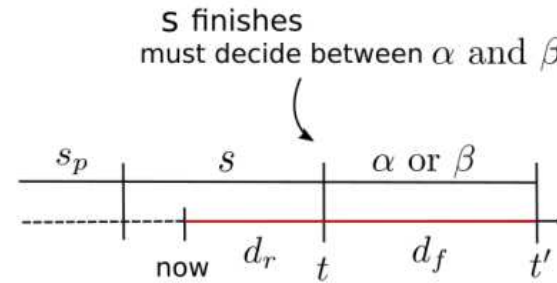
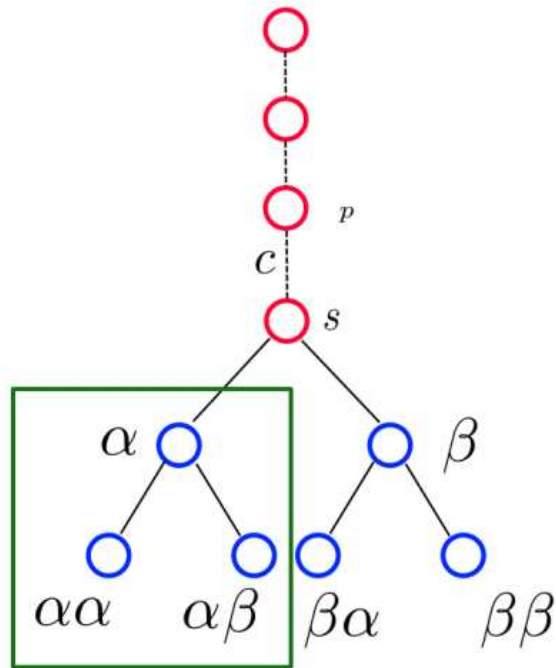
■ FACS Belief

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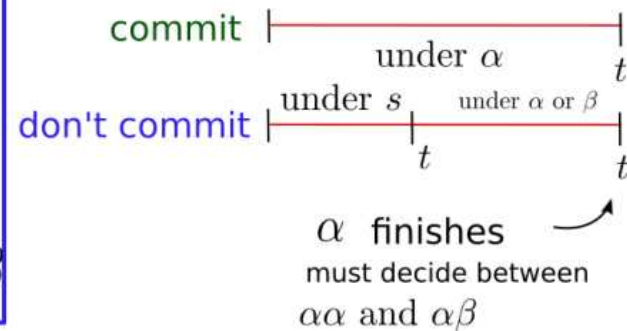
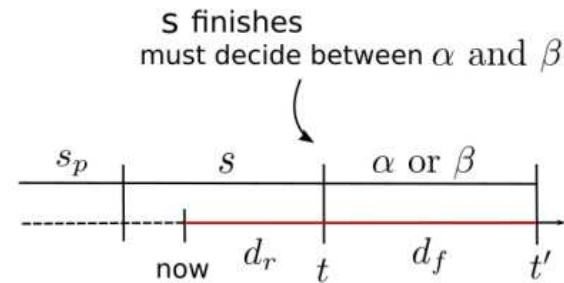
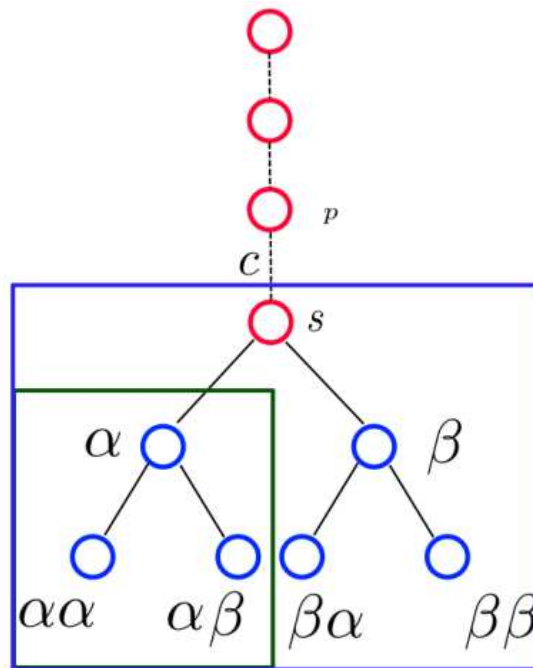
# Our Approach: Flexible Action Commitment Search (FACS)

we propose a principled way to make meta-level decision



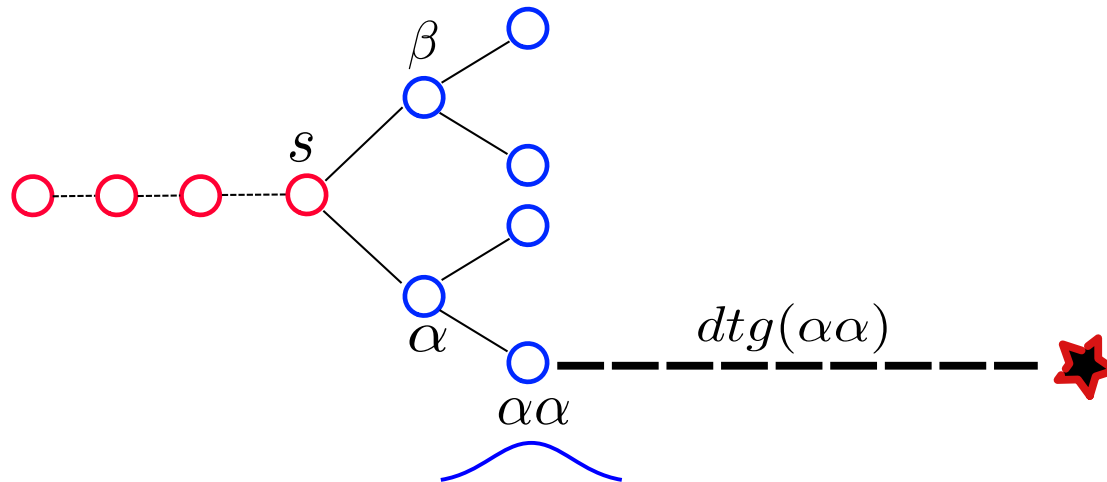
# Our Approach: Flexible Action Commitment Search (FACS)

we propose a principled way to make meta-level decision



# FACS: The Effect of Search

belief of where  $\hat{f}$  will be after search:



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# FACS: The Effect of Search

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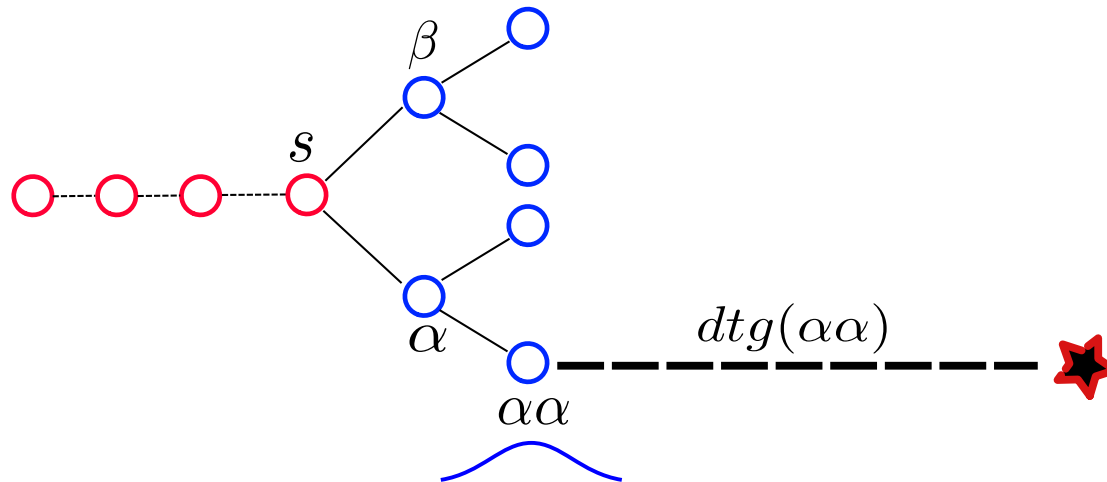
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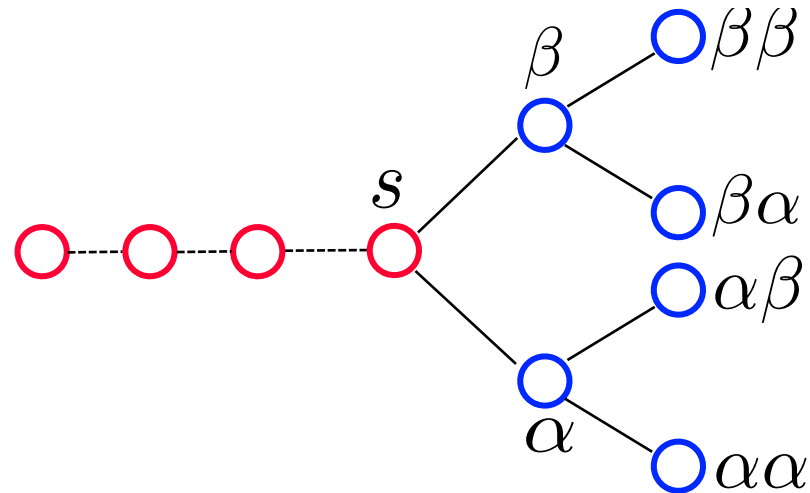
belief of where  $\hat{f}$  will be after search:



$$X_{\alpha\alpha}^d \sim \mathcal{N}(\hat{f}, \text{var}(\text{dtg}, d))$$

# FACS: Compute Utility

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$$U_{\text{commit}} = \mathbb{E} \left[ \min(X_{\alpha\alpha}^d, X_{\alpha\beta}^d) \right]$$

where  $d = (d_r + d_f)/2$

$$U_{\text{don't commit}} = P_{\text{choose } \alpha} \cdot U_{\alpha} + (1 - P_{\text{choose } \alpha}) \cdot U_{\beta}$$

commit when  $U_{\text{commit}}^{t'} > U_{\text{don't commit}}^{t'}$

# Synthetic Grid Pathfinding

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- Left: tar pit area → high cost for reckless committing
- Right: corridor area → need long lookahead to observe the local minima
- Middle: empty area → gain lookahead, no harm to commit